

Question 1: Solve

$$y'' + 4y' + 5y = 35e^{-4x}, \quad y(0) = -3, \quad y'(0) = 1$$

using the method of undetermined coefficients to find y_p .

Question 2: Find the general solution on $(0, \infty)$ to

$$x^2 y'' - 4xy' = x^5$$

using variation of parameters to find y_p .

Question 3: A mass of 1 kg stretches a spring $g/2$ m where $g = 9.8 \text{ m/s}^2$ is the usual gravitational constant. The stretched spring with 1 kg mass attached is allowed to come to rest at equilibrium. The mass travels through a medium which imparts a damping force equal to 3 times the velocity, and starting at time $t = 0$ s an external force of $f(t) = 2 \sin(t)$ is applied to the system. Find the equation of motion for the system for times $t \geq 0$.

(You may use any method you like to solve the resulting differential equation.)

Question 4: Solve using the Laplace transform:

$$y'' + y = \sqrt{2} \sin(\sqrt{2}t), \quad y(0) = 10, \quad y'(0) = 0$$

Question 5: Solve using the Laplace transform:

$$y' + y = f(t), \quad \text{where } y(0) = -1 \text{ and } f(t) = \begin{cases} 0, & 0 \leq t < 2 \\ e^t, & t \geq 2 \end{cases}$$