Question 1: For this question use the autonomous differential equation

$$\frac{dy}{dt} = (y-2)(e^{y-5}-1)$$

(i) Determine the equilibrium solutions

[2]

(ii) Sketch the one dimensional phase portrait.

[2]

(iii) Classify each equilibrium solution as stable, unstable or semi-stable

[2]

(iv) Sketch the equilibrium solution curves as well as approximate solution curves corresponding to $y(0) = 0$, $y(0) = 4$ and $y(0) = 7$:

[2]

(v) Use Euler's method with $h = 0.1$ and $y(0) = 4$ to approximate $y(0.2)$ to three decimal places.

[2]

Question 2: Solve the IVP $\frac{dy}{dx} = \frac{x(x^2+1)}{4y^3}$, $y(0) = \frac{-1}{\sqrt{2}}$. State your final answer in explicit form.

[5]

Question 3: Solve the IVP $(x+1)\frac{dy}{dx} + y = \ln(x)$, $y(1) = 10$, and give the largest interval $\mathcal{I}$ over which the solution is defined. State your final answer in explicit form.

[5]

Question 4: Find the general solution to $x\dfrac{dy}{dx} = 2xe^x - y + 6x^2$. You may leave your solution in implicit form.

[5]

Question 5: Find the general solution to $(y^2 + yx)dx - x^2dy = 0$ and also make note of any constant solutions. You may leave your solution in implicit form.

[5]

Question 6: Solve the IVP: $x^2\dfrac{dy}{dx} - 2xy = 3y^4$, $y(1) = 1/2$. You may leave your solution in implicit form.

[5]

Question 7: Solve the IVP: $\dfrac{dy}{dx} = \dfrac{1-x-y}{x+y}$, $y(0) = -2$. State your final answer in explicit form.

[5]