1. Solve using the Laplace transform:

$$y'' + 9y = 34e^{-5t} + 9\delta(t-5), \quad y(0) = 0, \quad y'(0) = -1$$

2. Solve using the Laplace transform:

$$y'' + 4y' + 5y = \delta(t-a), \quad y(0) = 0, \quad y'(0) = 0, \quad a > 0$$

3. For $x > 0$ the gamma function $\Gamma(x)$ is defined as

$$\Gamma(x) = \int_0^\infty r^{x-1} e^{-r}\, dr$$

(a) Show that $\Gamma(1) = 1$ and that $\Gamma(x+1) = x\Gamma(x)$. (This shows that the gamma function generalizes the factorial function, with $\Gamma(n+1) = n!$ for integer $n \geq 0$)

(b) Show that for $\alpha > -1$

$$\mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$$

(Hint: use the integral definition of the Laplace transform and let $r = st$.)

(c) Using methods from multivariable calculus (specifically, double integrals in polar coordinates) it can be shown that $\Gamma(1/2) = \sqrt{\pi}$ (you don't have to show this, just take it as fact.) Use this fact and parts (a) and (b) to determine

$$\mathcal{L}\{t^{-1/2} + 2t^{3/2} + 8t^{5/2}\}$$

Your final answer should not contain gamma functions.

4.

$$\begin{aligned} x_1'(t) &= x_1(t) + 3x_2(t) \\ x_2'(t) &= 5x_1(t) + 3x_2(t) \end{aligned}$$

5.

$$\begin{aligned} x_1'(t) &= 6x_1(t) - x_2(t) \\ x_2'(t) &= 5x_1(t) + 2x_2(t) \end{aligned}$$

6.

$$\begin{aligned} x_1'(t) &= 2x_1(t) + 4x_2(t) \quad , \quad x_1(0) = -1 \\ x_2'(t) &= -x_1(t) + 6x_2(t) \quad , \quad x_2(0) = 6 \end{aligned}$$