Question 1: Find and simplify $f(a+2)$ if $f(x) = -x^2 + 4x + 1$.

$$f(a+2) = -(a+2)^2 + 4(a+2) + 1$$

$$= -a^2 \cancel{-4a} - 4 + \cancel{4a} + 8 + 1$$

$$= \boxed{-a^2 + 5}$$

[3]

Question 2:

(i) Find the equation of the line through the points $(5, -3)$ and $(1, -7)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-7)}{5-1} = \frac{4}{4} = 1$$

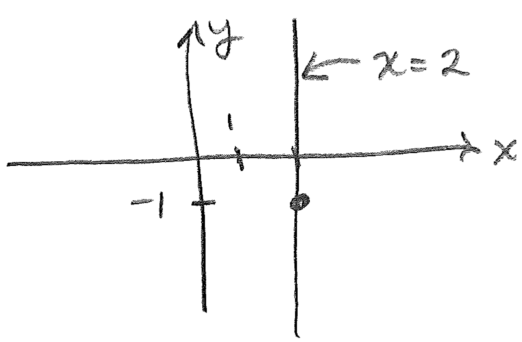
$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 1(x-5)$$

$$\boxed{y + 3 = x - 5} \quad \text{or} \quad \boxed{y = x - 8}$$

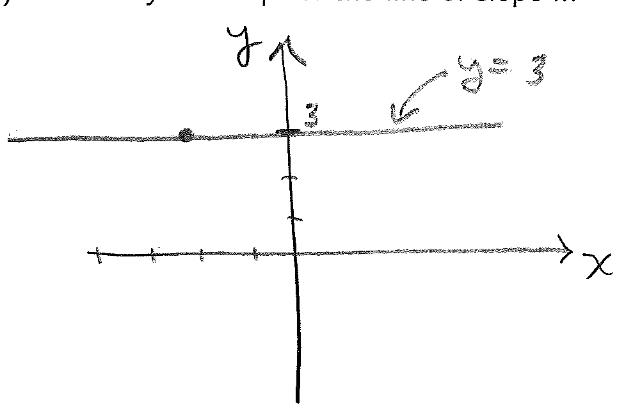
[3]

(ii) Graph the line through the point $(2, -1)$ having undefined slope.



[2]

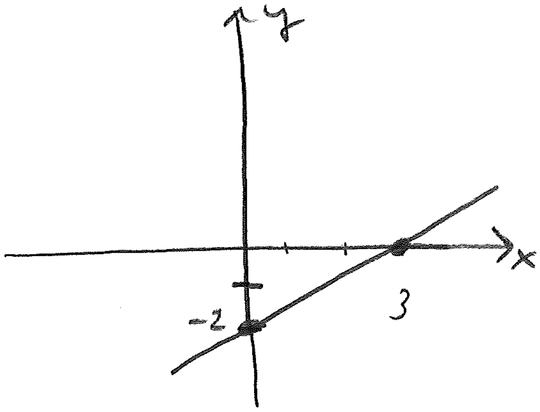
(iii) Find the y-intercept of the line of slope $m = 0$ through the point $(-7/3, 3)$.



y-intercept : $\boxed{(0, 3)}$

[2]

Question 3: Find the equation of the line having x-intercept $(3, 0)$ and y-intercept $(0, -2)$.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{0-3} = \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 0 = \frac{2}{3}(x-3)}$$

or $\boxed{y = \frac{2}{3}x - 2}$

[3]

Question 4: Find an equation of the line through the point $(1, 6)$ that is perpendicular to the line $3x + 5y = 1$.

$$3x + 5y = 1$$

$$\Rightarrow y = -\frac{3}{5}x + \frac{1}{5}$$

$$m_1 = -\frac{3}{5}$$

So slope of line we want is $m = \frac{-1}{m_1} = \frac{5}{3}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 6 = \frac{5}{3}(x-1)}$$

or $\boxed{y = \frac{5}{3}x + \frac{13}{3}}$

[3]

Question 5: Find the value of k so that the line through $(3, -2)$ and $(k, 3)$ is parallel to the line $3y + 2x = 6$.

$$3y + 2x = 6.$$

$$\Rightarrow y = -\frac{2}{3}x + 2$$

$$m_1 = -\frac{2}{3}$$

So we need $-\frac{2}{3} = \frac{3-(-2)}{k-3}$

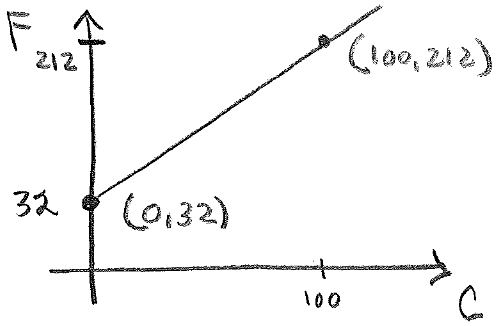
$$-2k + 6 = 15$$

$$k = \frac{15-6}{-2}$$

$$k = \boxed{-\frac{9}{2}}$$

[4]

Question 6: The relationship between temperature expressed in degrees Celsius (°C) and degrees Fahrenheit (°F) is linear. A temperature of 0°C corresponds to 32°F, while a temperature of 100°C corresponds to 212°F. Using this information, and letting C represent temperature measured in Celsius and F temperature measured in degrees Fahrenheit, express F as a function of C.



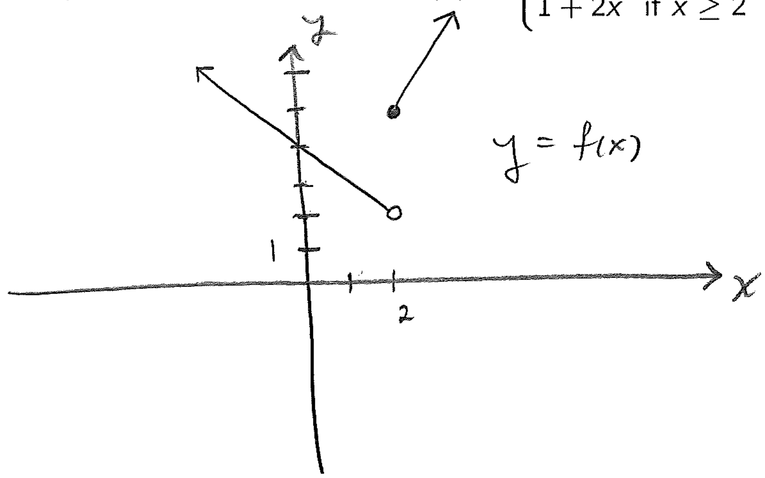
$$m = \frac{212-32}{100-0} = \frac{9}{5}$$

$$F - 32 = \frac{9}{5}(C-0)$$

$$\boxed{F = \frac{9}{5}C + 32}$$

[4]

Question 7: Sketch the piecewise defined function $f(x) = \begin{cases} 4-x & \text{if } x < 2 \\ 1+2x & \text{if } x \geq 2 \end{cases}$.



[4]

Question 8: What value of k will make the function $f(x)$ continuous if $f(x) = \begin{cases} k-x & \text{if } x < 2 \\ 1+2x & \text{if } x \geq 2 \end{cases}$.

At $x = 2$ we require

$$k - x = 1 + 2x$$

$$\Rightarrow k - 2 = 1 + 2(2)$$

$$k = 1 + 4 + 2$$

$$\boxed{k = 7}$$

[2]

Question 9: Let $f(x) = \sqrt{4x+1}$ and $g(x) = 1/x$. Find and simplify $(\frac{f}{g})(x)$ and state the domain.

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{\sqrt{4x+1}}{\frac{1}{x}} \quad \begin{array}{l} \} \; x \geq -\frac{1}{4} \\ \} \; x \neq 0 \end{array}$$

$$= \boxed{x\sqrt{4x+1}.}$$

$\therefore$ domain is $[-\frac{1}{4}, 0) \cup (0, \infty)$.

[3]

Question 10: Let $f(x) = 1/x$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\left(\frac{1}{x+h}\right) - \left(\frac{1}{x}\right)}{h}$$

$$= \frac{1}{h}\left[\frac{x - x - h}{(x+h)x}\right]$$

$$= \frac{-\cancel{h}}{\cancel{h}(x+h)x}$$

$$= \boxed{\frac{-1}{(x+h)x}}$$

[5]

Question 11: Let $h(x) = \sqrt{6x+1} - 12$. Find functions f and g such that $(f \circ g)(x) = h(x)$. Do not let $g(x) = x$.

$$h(x) = \sqrt{6x+1} - 12 = f(g(x))$$

Let $g(x) = 6x+1$, $f(x) = \sqrt{x} - 12$

or $g(x) = \sqrt{6x+1}$, $f(x) = x - 12$

There are other correct answers.

[2]

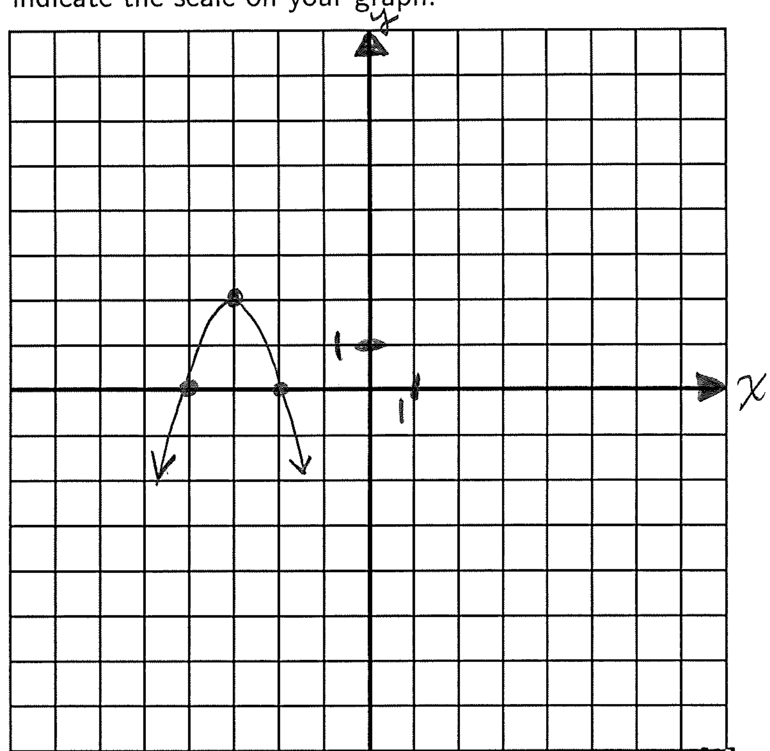
Question 12: For this question use the function $f(x) = -2x^2 - 12x - 16$.

(i) Graph the function. Draw the x and y axes and indicate the scale on your graph.

$f(x) = -2\left[x^2+6x\right] - 16$

$= -2\left[(x+3)^2 - 9\right] - 16$

$= -2(x+3)^2 + 2$



[3]

(ii) State the vertex and range of $f(x)$. $\boxed{\text{Vertex } (-3,2),\ \text{range } (-\infty, 2]}$ [1]

(iii) Find the x-intercepts of the graph, if any.

$-2(x+3)^2 + 2 = 0$

$(x+3)^2 = \frac{-2}{-2} = 1$

$(x+3) = \pm 1$

$x = -3 \pm 1$

$\boxed{x = -2, -4}$

[2]

Question 13: An object projected upward has height above the ground given by $s(t) = -16t^2 + 64t + 100$ where t represents time in seconds and the height is measured in feet. Determine the maximum height reached by the object.

$y = -16t^2 + 64t + 100$

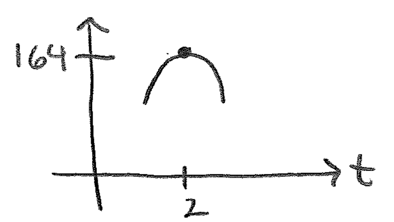
$= -16\left[t^2 - 4t\right] + 100$

$= -16\left[(t-2)^2 - 4\right] + 100$

$= -16(t-2)^2 + 164$

Vertex at $(2, 164)$

graph is



$\boxed{\therefore \text{ maximum height is 164 feet}}$

[4]