Question 1: Solve for x:

$$\frac{2x+5}{2} - \frac{3x}{x-2} = x \quad \Big\} \; x \neq 2$$

$$(x-2)(2x+5) - (2)(3x) = 2(x-2)x$$

$$\cancel{2x^2} + x - 10 - 6x = \cancel{2x^2} - 4x$$

$$-x - 10 = 0$$

$$\boxed{x = -10}$$

(See 1.6.17).

Check:

$$\frac{2(-10)+5}{2} - \frac{3(-10)}{-10-2} \stackrel{?}{=} -10$$

$$-\frac{15}{2} - \frac{30}{12} \stackrel{?}{=} -10$$

$$-10 = -10 \checkmark$$

[5]

Question 2: Solve for x:

$$\frac{2x}{x-2} = 5 + \frac{4x^2}{x-2} \quad \Big\} \; x \neq +2$$

$$2x = 5(x-2) + 4x^2$$

$$2x = 5x - 10 + 4x^2$$

$$4x^2 + 3x - 10 = 0$$

$$4x^2 - 5x + 8x - 10 = 0$$

$$x(4x-5) + 2(4x-5) = 0$$

$$(4x-5)(x+2) = 0$$

(See 1.6.35)

$$\boxed{x = \frac{5}{4}}, \quad \boxed{x = -2}$$

Check: $\frac{2(\frac{5}{4})}{\frac{5}{4}-2} \stackrel{?}{=} 5 + \frac{4(5/4)^2}{5/4 - 2}$; $-\frac{10}{3} = -\frac{10}{3} \checkmark$.

[5]

$\frac{2(-2)}{-2-2} \stackrel{?}{=} 5 + \frac{4(-2)^2}{-2-2}$; $1 \stackrel{?}{=} 1 \checkmark$

Question 3: Solve for x:

$$\sqrt{4x+5} - 6 = 2x - 11$$

$$\left(\sqrt{4x+5}\right)^2 = (2x-5)^2$$

$$4x+5 = 4x^2 - 20x + 25$$

$$4x^2 - 24x + 20 = 0$$

$$x^2 - 6x + 5 = 0$$

$$(x-5)(x-1) = 0$$

$$\boxed{x = 5}, \ \cancel{x=1}$$

Check:

$x=5$: $\sqrt{4(5)+5} - 6 \stackrel{?}{=} 2(5) - 11$

$-1 = -1$ ✓

$x=1$: $\sqrt{4(1)+5} - 6 \stackrel{?}{=} 2(1) - 11$

$-3 \stackrel{?}{=} -9$ ✗

[5]

Question 4: Solve the inequality. State the answer using interval notation

$$\frac{4x+7}{-3} \le 2x + 5$$

note!

$$4x+7 \ge -3(2x+5)$$

$$4x+7 \ge -6x - 15$$

$$10x \ge -22$$

$$x \ge \frac{-22}{10}$$

$$x \ge \frac{-11}{5}$$

$$\boxed{\left[\frac{-11}{5}, \infty\right)}$$

(see 1.7.21)

[5]

Question 5: Solve the inequality. State the answer using interval notation

$$-4 \leq \frac{x+1}{2} \leq 5$$

$-8 \leq x+1 \leq 10$

$-9 \leq x \leq 9$

$\boxed{[-9, 9]}$

(see 1.7.35)

[5]

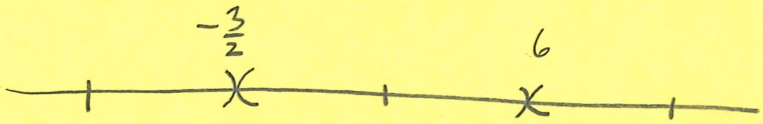
Question 6: Solve the inequality. State the answer using interval notation

$$2x^2 - 9x < 18$$

$2x^2 - 9x - 18 < 0$

$2x^2 - 9x - 18 = 0$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-18)}}{2(2)} = \frac{9 \pm 15}{4} = 6, -\frac{3}{2}$$



test: -2 0 7

$2x^2 - 9x - 18 \overset{?}{<} 0$: F T F

$\boxed{\left(-\frac{3}{2}, 6\right)}$

(see 1.7.41)

[5]

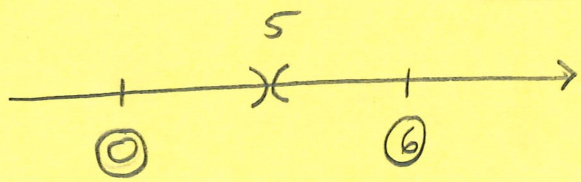
Question 7: Solve the inequality. State the answer using interval notation

$$\frac{x+3}{x-5} \le 1 \quad \Big\} \quad x \neq 5 \text{ in solution set.}$$

$$\frac{x+3}{x-5} - 1 \le 0$$

$$\frac{x+3-(x-5)}{x-5} \le 0$$

$$\left.\frac{8}{x-5} \le 0 \quad\right\} \quad x-5=0 \text{ at } x=5$$



test:

$$\frac{8}{x-5} \overset{?}{\le} 0 : \quad T \qquad F$$

$$\boxed{(-\infty, 5)}$$

(see 1.7.71)

[5]

Question 8: A line segment has midpoint $P(12, 6)$ and one endpoint $Q(19, 16)$. Determine the other endpoint.

Let (x_2, y_2) be the other endpoint.

Then $\left(\frac{19+x_1}{2}, \frac{16+y_1}{2}\right) = (12, 6)$

$$\therefore \frac{19+x_1}{2} = 12 \ ; \ \frac{16+y_1}{2} = 6$$

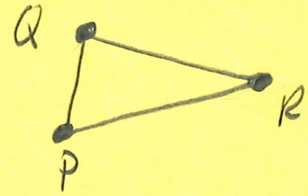
$$x_1 = 5 \ ; \ y_1 = -4$$

So the other endpoint is $\boxed{(5, -4)}$ (see 2.1.37)

[5]

Question 9: Determine whether the three points are vertices of a right triangle: $P(-2,-5)$, $Q(1,7)$, $R(3,15)$.

$$d(P,Q) = \sqrt{(1-(-2))^2+(7-(-5))^2} = \sqrt{153}$$



$$d(Q,R) = \sqrt{(3-1)^2+(15-7)^2} = \sqrt{68}$$

$$d(R,P) = \sqrt{(3-(-2))^2+(15-(-5))^2} = \sqrt{425}$$

$$(\sqrt{153})^2 + (\sqrt{68})^2 \neq (\sqrt{425})^2,$$

so the given points are $\boxed{\text{not}}$ the vertices of a right triangle.

(see 2.1.27)

[5]

Question 10: Write the following equation of a circle in center-radius form $(x-h)^2+(y-k)^2=r^2$ and state the center and radius:

$$x^2+y^2-12x+10y+25=0$$

$$x^2-12x \quad + \quad y^2+10y \quad +25=0$$

$$(x-6)^2-36+(y+5)^2-\cancel{25}+\cancel{25}=0$$

$$(x-6)^2+(y+5)^2=36.$$

$$\boxed{\therefore \text{ Center } (h,k)=(6,-5), \text{ radius } r=6.}$$

(see 2.2.27 → 35)

[5]