

Question 1:

(a) Simplify, assuming all variables represent positive real numbers:

$$\begin{aligned} & \sqrt{24m^5n^6} \\ &= \sqrt{(4)(6)(m^2)^2(m)(n^3)^2} \quad (\text{see R.7.59}) \\ &= 2m^2n^3\sqrt{6m} \end{aligned}$$

[2]

(b) Simplify, assuming all variables represent positive real numbers:

$$\begin{aligned} & \sqrt[4]{\frac{32x^5}{y^8}} \quad (\text{see R.7.69}) \\ &= \sqrt[4]{\frac{(2^4)(2)(x^4)(x)}{(y^2)^4}} \\ &= \frac{2x\sqrt{2x}}{y^2} \end{aligned}$$

[2]

(c) Simplify, assuming all variables represent positive real numbers:

$$\begin{aligned} & \frac{\sqrt[3]{mn}\sqrt[3]{m^2}}{\sqrt[3]{n^2}} \\ &= \sqrt[3]{\frac{m^3n}{n^2}} \\ &= \frac{m}{\sqrt[3]{n}} \cdot \frac{\sqrt[3]{n^2}}{\sqrt[3]{n^2}} \end{aligned}$$

$$= \frac{m\sqrt[3]{n^2}}{n} \quad (\text{see R.7.93})$$

[2]

(d) Rationalize the denominator:

$$\begin{aligned} & \frac{2a}{3+\sqrt{a+b}} \cdot \frac{3-\sqrt{a+b}}{3-\sqrt{a+b}} \\ &= \frac{2a(3-\sqrt{a+b})}{9-(a+b)} \quad (\text{see R.7.107}) \\ &= \frac{6a-2a\sqrt{a+b}}{9-a-b} \end{aligned}$$

[4]

Question 2:

(a) Simplify and express your final answer without negative exponents:

$$\frac{(7v^2)^{-4}(7v^5)^{-2}}{(7^{-3}v^{-4})^2} \quad (\text{See R.6.45})$$

$$= \frac{\cancel{(7^{-4})} \cancel{(v^{-8})} \cancel{(7^{-2})} (v^{-10})}{\cancel{(7^{-6})} \cancel{(v^{-8})}}$$

$$= \frac{1}{v^{-10}}$$

[3]

(b) Expand and simplify:

$$(2z^{1/2} - z)(z^{1/2} + z) \quad (\text{See R.6.85})$$

$$= 2z - z^{3/2} + 2z^{3/2} - z^2$$

$$= \boxed{2z + z^{3/2} - z^2}$$

[3]

(c) Simplify and express your final answer without negative exponents:

$$\frac{2(2x-3)^{1/3} - (x-1)(2x-3)^{-2/3}}{(2x-3)^{2/3}} \cdot \frac{(2x-3)^{2/3}}{(2x-3)^{2/3}}$$

$$= \frac{2(2x-3)^1 - (x-1)(2x-3)^1}{(2x-3)^{4/3}} \quad (\text{see R.6.115})$$

$$= \boxed{\frac{3x-5}{(2x-3)^{4/3}}}$$

[4]

Question 3:

(a) Solve for x :

$$5(x+3) + 4x - 3 = -(2x-4) + 2$$

$$5x+15+4x-3 = -2x+4+2$$

$$11x = -6 \quad (\text{see 1.1.17}).$$

$$x = \frac{-6}{11}$$

[2]

(b) Solve for w where

$$S = 2lw + 2wh + 2hl \quad (\text{see 1.1.47})$$

$$S = w(2l+2h) + 2hl$$

$$w = \frac{S-2hl}{2l+2h}$$

[2]

(c) An 18 month investment at 2.4% simple interest earned \$126 in interest. What was the original amount invested? Round your final answer to the nearest dollar.

$$t = 1.5 \quad I = Prt \quad (\text{see Supplementary Exercises 1.1.4})$$

$$r = 2.4\% = 0.024 \quad P = \frac{I}{rt}$$

$$I = \$126 \quad P = \frac{126}{(0.024)(1.5)}$$

$$P = \$3500$$

[3]

(d) A person borrows \$300 from a payday loan company and must repay \$345 fourteen days later. Determine the simple rate of interest being charged. State your answer as a percentage rounded to one decimal place.

$$t = \frac{14}{365}$$

$$P = \$300$$

$$I = 345 - 300 = \$45$$

$$r = \frac{I}{Pt} = \frac{45}{(300)\left(\frac{14}{365}\right)} \approx 3.9107$$

$$r = 391.1\%$$

Alternatively, using $t = \frac{2 \text{ weeks}}{52 \frac{\text{weeks}}{\text{yr}}} = \frac{1}{26} \text{ yr}$, $r \approx 390\%$ is also correct.

[3]

Question 4:

- (a) A person invests some money at 2.5% simple interest and twice as much as the first amount at 3%. After one year the total interest earned from both investments is \$850. How much was invested at the 3% rate? Round your answer to the nearest dollar.

Let x = amount invested at 3%

so $\frac{x}{2}$ = amount invested at 2.5%. (see 1.2.37).

$$\therefore \left(\frac{x}{2}\right)(0.025) + (x)(0.03) = \$850$$

$$x \left[\frac{0.025}{2} + 0.03 \right] = 850$$

$$x [0.0425] = 850$$

$$x = \frac{850}{0.0425} = \$20,000$$

\therefore \$20,000 was invested at 3%.

[5]

- (b) A boat driving on a lake with no current travels at a constant speed. The same boat driving on a river against the current takes 20 minutes to reach its destination. The return trip downriver with the current takes only 15 minutes. If the river current is 5 km/h what is the boat speed when there is no current?

Let v = speed with no current. (see 1.2.19 & 21).

Going: speed = $v - 5$
time = $\frac{20}{60} = \frac{1}{3}$ hr.

Return: speed = $v + 5$
time = $\frac{15}{60} = \frac{1}{4}$ hr.

$$\therefore (v-5)\left(\frac{1}{3}\right) = (v+5)\left(\frac{1}{4}\right)$$

$$4v - 20 = 3v + 15$$

$$v = 35$$

\therefore Boat speed with no current is $35 \frac{\text{km}}{\text{hr}}$

[5]

Question 5:

(a) Find all solutions to

$$x^2 + 2x - 8 = 0$$

(see 1.4.13)

$$(x-2)(x+4) = 0$$

$$\boxed{x=2, x=-4}$$

[3]

(b) Find all solutions to

$$-6x^2 = 3x - 2$$

(see 1.4.58 & 59).

$$6x^2 + 3x - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(6)(-2)}}{2(6)}$$

$$\boxed{x = \frac{-3 \pm \sqrt{57}}{12}}$$

[3]

(c) One square has side length 5 cm longer than another and area 95 cm² larger than the other. Determine the side length of the smaller square.Let x = side length of smaller square $\therefore x+5$ = side length of larger square.

$$(x+5)^2 = x^2 + 95$$

$$\cancel{x^2} + 10x + 25 = \cancel{x^2} + 95$$

$$10x = 70$$

$$x = 7$$

$$\therefore \text{The smaller square has side length 7 cm.}$$

(see 1.5.21 & 22).

[4]