Question 1:

(a) Simplify and express your final answer without negative exponents:

$$\frac{(6u^2)^{-4}(6u^5)^{-2}}{(6^{-3}u^{-4})^2}$$

(See R.6.45)

$$= \frac{6^{-4}u^{-8}6^{-2}u^{-10}}{6^{6}u^{-8}}$$

$$= \boxed{\frac{1}{u^{10}}}$$

[3]

(b) Expand and simplify:

$$(2z^{1/2}+z)(z^{1/2}-z)$$

(see R.6.85)

$$= 2z + z^{3/2} - 2z^{3/2} - z^2$$

$$= \boxed{2z - z^{3/2} - z^2}$$

[3]

(c) Simplify and express your final answer without negative exponents:

$$\frac{2(2x-3)^{1/3} - (x-1)(2x-3)^{-2/3}}{(2x-3)^{2/3}} \cdot \frac{(2x-3)^{2/3}}{(2x-3)^{2/3}}$$

$$= \frac{2(2x-3)^{1} - (x-1)(2x-3)^{0}}{(2x-3)^{4/3}} \quad | \quad (2x-3)^{0} = 1$$

$$= \frac{4x-6-x+1}{(2x-3)^{4/3}}$$

(see R.6.115).

$$= \boxed{\frac{3x-5}{(2x-3)^{4/3}}}$$

[4]

Question 2:

(a) Simplify, assuming all variables represent positive real numbers:

(see R.7.59)

$$\sqrt{24m^6n^5}$$

$$= \sqrt{(4)(6)(m^3)^2(n^2)^2(n)}$$

$$= \boxed{2m^3n^2\sqrt{6n}}$$

[2]

(b) Simplify, assuming all variables represent positive real numbers:

(see R.7.69)

$$\sqrt[4]{\frac{32x^5}{y^8}}$$

$$= \sqrt[4]{\frac{(2^4)(2)(x^4)(x)}{(y^2)^4}}$$

$$= \boxed{\frac{2x\sqrt[4]{2x}}{y^2}}$$

[2]

(c) Simplify, assuming all variables represent positive real numbers:

$$\frac{\sqrt[3]{mn}\,\sqrt[3]{m^2}}{\sqrt[3]{n^2}}$$

$$= \sqrt[3]{\frac{m^3n}{n^2}}$$

$$= \frac{m}{\sqrt[3]{n}} \cdot \frac{\sqrt[3]{n^2}}{\sqrt[3]{n^2}}$$

$$= \boxed{\frac{m\sqrt[3]{n^2}}{n}}$$

(see R.7.93)

[2]

(d) Rationalize the denominator:

$$\frac{3a}{2+\sqrt{a+b}} \cdot \frac{2-\sqrt{a+b}}{2-\sqrt{a+b}}$$

$$= \frac{3a(2-\sqrt{a+b})}{4-(a+b)}$$

(see R.7.107)

$$= \boxed{\frac{6a-3a\sqrt{a+b}}{4-a-b.}}$$

[4]

Question 3:

(a) Solve for x:

$5(x+3)+4x-2=-(2x-4)+3$ (see 1.1.17)

$5x+15+4x-2=-2x+4+3$

$11x = -6$

$x = \frac{-6}{11}$

[2]

(b) Solve for h where

$S = 2\ell w + 2wh + 2h\ell$ (see 1.1.47)

$S = 2\ell w + h(2w+2\ell)$

$h = \frac{S-2\ell w}{2w+2\ell}$

[2]

(c) An 18 month investment at 2.4% simple interest earned \$108 in interest. What was the original amount invested? Round your final answer to the nearest dollar.

$t = 1.5$

$r = 2.4\% = 0.024$

$I = \$108$

$I = Prt$

$P = \frac{I}{rt}$

(see Supplementary Exercises 1.1.4)

$P = \frac{108}{(0.024)(1.5)}$

$P = \$3000$

[3]

(d) A person borrows \$300 from a payday loan company and must repay \$345 fourteen days later. Determine the simple rate of interest being charged. State your answer as a percentage rounded to one decimal place.

$t = \frac{14}{365}$

$P = \$300$

$I = 345 - 300 = \$45$

$r = \frac{I}{Pt}$

$= \frac{45}{(300)\left(\frac{14}{365}\right)}$

$\doteq 3.9107$

$= 391.1\%$

Alternatively, using $t = \frac{2 \text{ weeks}}{52 \frac{\text{weeks}}{\text{yr}}} = \frac{1}{26}$ yr,

$r \doteq 390\%$ also correct.

[3]

Question 4:

(a) A person invests some money at 2.5% simple interest and twice as much as the first amount at 3%. After one year the total interest earned from both investments is $850. How much was invested at the 3% rate? Round your answer to the nearest dollar.

Let x = amount invested at 3%

$\therefore$ $\frac{x}{2}$ = amount invested at 2.5%. (see 1.2.37)

So $\left(\frac{x}{2}\right)(0.025)(1) + (x)(0.03)(1) = 850$

$$x\left[\frac{0.025}{2} + 0.03\right] = 850$$

$$x[0.0425] = 850$$

$$x = \frac{850}{0.0425} = \$20{,}000$$

$20,000 was invested at 3%.

[5]

(b) A boat driving on a lake with no current travels at a constant speed. The same boat driving on a river against the current takes 20 minutes to reach its destination. The return trip downriver with the current takes only 15 minutes. If the river current is 5 km/h what is the boat speed when there is no current?

Let r = speed with no current. (see 1.2.19 & 21).

Going: speed = $r-5$

time = $\frac{20}{60} = \frac{1}{3}$ hr.

Return: speed = $r+5$

time = $\frac{15}{60} = \frac{1}{4}$ hr.

$\therefore$ $(r-5)\left(\frac{1}{3}\right) = (r+5)\left(\frac{1}{4}\right)$

$$4r - 20 = 3r + 15$$

$$r = 35$$

$\therefore$ Boat speed with no current is $35 \frac{km}{hr}$

[5]

Question 5:

(a) Find all solutions to

$$x^2 + 2x - 8 = 0$$

(see 1.4.13)

$$(x-2)(x+4)=0$$

$$\boxed{x=2, \quad x=-4}$$

[3]

(b) Find all solutions to

$$-6x^2 = 3x - 2$$

$$6x^2+3x-2=0$$

(see 1.4.58 & 59)

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2-4(6)(-2)}}{2(6)}$$

$$= \boxed{\frac{-3 \pm \sqrt{57}}{12}}$$

[3]

(c) One square has side length 5 cm longer than another and area 95 cm^2 larger than the other. Determine the side length of the larger square.

Let x = side length of larger square.

$\therefore$ $x-5$ = side length of smaller square.

$$(x-5)^2 = x^2 - 95$$

$$x^2 - 10x + 25 = x^2 - 95$$

$$10x = 120$$

$$\boxed{x = 12}$$

(see 1.5.21 & 22).

$\therefore$ The larger square has side length 12 cm.

[4]