

Question 1: For this question use the following sets:

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}, \quad M = \{0, 2, 4, 6, 8\}, \quad N = \{1, 3, 5, 7, 9, 11, 13\},$$

$$Q = \{0, 2, 4, 6, 8, 10, 12\}, \quad R = \{0, 1, 2, 3, 4\}$$

Determine the following:

(a) $M \cap R = \boxed{\{0, 2, 4\}}$ } homework R.1.81

(b) $M \cup R = \boxed{\{0, 1, 2, 3, 4, 6, 8\}}$ } R.1.84 [2]

(c) $M' \cap N = \{1, 3, 5, 7, 9, 10, 11, 12, 13\} \cap \{1, 3, 5, 7, 9, 11, 13\}$
 $= \boxed{\{1, 3, 5, 7, 9, 11, 13\}}$ } See homework R.1.91 [2]

(d) $(M' \cup Q) \cap R = (\{1, 3, 5, 7, 9, 10, 11, 12, 13\} \cup \{0, 2, 4, 6, 8, 10, 12\}) \cap \{0, 1, 2, 3, 4\}$
 homework
 R.1.101 } $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\} \cap \{0, 1, 2, 3, 4\}$
 $= \boxed{\{0, 1, 2, 3, 4\}}$ [3]

Question 2:

- (a) Let $p = -4$, $q = 8$ and $r = -10$. Evaluate $\frac{\left(\frac{q}{2} - \frac{r}{3}\right)}{\left(\frac{3p}{4} + \frac{q}{8}\right)}$. } homework R.2.43

$$\frac{\left(\frac{8}{2} - \frac{-10}{3}\right)}{\left(\frac{3(-4)}{4} + \frac{8}{8}\right)} = \frac{\left(\frac{8}{2} - \frac{-10}{3}\right)}{\left(\frac{(-3)(-4)}{4} + \frac{8}{8}\right)} = \frac{\left(\frac{4}{1} + \frac{10}{3}\right)}{(3+1)} = \frac{\left(\frac{12+10}{3}\right)}{\left(\frac{4}{1}\right)} = \left(\frac{22}{3}\right)\left(\frac{1}{4}\right) = \boxed{\frac{11}{6}}$$

[4]

- (b) Simplify $-\frac{1}{4}(20m + 8y - 32z)$. } R.2.68

$$\begin{aligned} &= \left(-\frac{1}{4}\right)(20m) + \left(-\frac{1}{4}\right)(8y) + \left(-\frac{1}{4}\right)(-32z) \\ &= \boxed{-5m - 2y + 8z} \end{aligned}$$

[3]

- (c) Let $x = -4$ and $y = 2$. Evaluate $\frac{|x| + 2|y|}{-|xy|}$. } see homework R.2.95

$$\frac{|x| + 2|y|}{-|xy|} = \frac{|-4| + 2|2|}{-|(-4)(2)|} = \frac{4 + 4}{-|-8|} = \frac{8}{-8} = \boxed{-1}$$

[3]

Question 3:

(a) Simplify $(-4xy^3)(7x^2y) = \boxed{-28x^3y^4}$ } R.3.20
 (see homework R.3.17 and R.3.21)

[2]

(b) Expand (that is, find the product and simplify): $(3w + 2)(-w^2 + 4w - 3)$.

R.3.62
 (see homework R.3.61)

$$\left. \begin{aligned} &= -3w^3 + 12w^2 - 9w - 2w^2 + 8w - 6 \\ &= \boxed{-3w^3 + 10w^2 - w - 6} \end{aligned} \right\}$$

[3]

(c) Perform the division: $\frac{x^4 + 5x^2 + 5x + 27}{x^2 + 3} = \boxed{(x^2 + 2) + \frac{(5x + 21)}{x^2 + 3}}$

$$\begin{array}{r} x^2 + 2 \\ x^2 + 0x + 3 \overline{) x^4 + 0x^3 + 5x^2 + 5x + 27} \\ \underline{-(x^4 + 0x^3 + 3x^2)} \\ 2x^2 + 5x + 27 \\ \underline{-(2x^2 + 0x + 6)} \\ 5x + 21 \end{array}$$

} 50

(See homework R.3.103)

[5]

Question 4:

(a) Factor completely: $(10ab - 6b) + (35a - 21)$

$$= 2b(5a-3) + 7(5a-3)$$

$$= \boxed{(5a-3)(2b+7)}$$

} R.4.30
(See homework R.4.29)

[2]

(b) Factor completely: $36x^3 + 18x^2 - 4x$

$$= 2x[18x^2 + 9x - 2]$$

$$(18)(-2) = -36 = (12)(-3)$$

$$= 2x[18x^2 - 3x + 12x - 2]$$

$$= 2x[3x(6x-1) + 2(6x-1)]$$

$$= \boxed{2x(6x-1)(3x+2)}$$

} R.4.42
(See homework
R.4.37 and R.4.39)

[2]

(c) Factor completely: $(x-4)^3 + 64$

$$= (x-4)^3 + 4^3 \quad \left. \begin{array}{l} \text{sum of cubes,} \\ \end{array} \right\} \text{R.4.88}$$

(See homework
R.4.81)

$$= [(x-4) + 4][(x-4)^2 - 4(x-4) + 4^2]$$

$$= x[x^2 - 8x + 16 - 4x + 16 + 16]$$

$$= \boxed{x(x^2 - 12x + 48)}$$

[3]

(d) Factor completely: $a^4 - 3a^2 - 54$

Let $w = a^2$:

$$w^2 - 3w - 54$$

$$= w^2 - 9w + 6w - 54$$

$$= w(w-9) + 6(w-9)$$

$$= (w+6)(w-9)$$

$$= (a^2+6)(a^2-9)$$

} difference of squares

$$= \boxed{(a^2+6)(a-3)(a+3)}$$

(See in-class example
of Wed, Jan 16.)

[3]

Question 5:

(a) Find the following product and express your answer in lowest terms:

$$\frac{6x-18}{9x^2+6x-24} \cdot \frac{12x-16}{4x-12}$$

$$= \frac{\cancel{6}(x-3)\cancel{4}(3x-4)}{\cancel{3}(x+2)\cancel{3}\cancel{4}(x-3)}$$

$$= \boxed{\frac{2}{x+2}}$$

$$\begin{aligned} & \text{For} \\ & 9x^2 + 6x - 24 \\ & = 3(3x^2 + 2x - 8) \\ & \quad \underbrace{(3)(-8) = -24} \\ & \quad \quad = (6)(-4) \\ & = 3(3x^2 + 6x - 4x - 8) \\ & = 3[3x(x+2) - 4(x+2)] \\ & = 3(x+2)(3x-4) \end{aligned}$$

} R.5.40
 (see homework R.5.41)

[5]

(b) Perform the following subtraction and express your answer in lowest terms:

$$\frac{3x}{x^2+x-12} - \frac{x}{x^2-16}$$

$$= \frac{3x}{(x+4)(x-3)} - \frac{x}{(x-4)(x+4)}$$

$$= \frac{3x(x-4) - x(x-3)}{(x+4)(x-3)(x-4)}$$

$$= \frac{3x^2 - 12x - x^2 + 3x}{(x+4)(x-3)(x-4)}$$

$$= \frac{2x^2 - 9x}{(x+4)(x-3)(x-4)}$$

$$= \boxed{\frac{x(2x-9)}{(x+4)(x-3)(x-4)}}$$

} R.6.69
 (see homework R.6.67)

[5]