## Math 371 Test 2 Focus Topics

For Test 2 you should be familiar with all homework problems assigned in Asn 5, 6, 7 & 8 as well as the theory covered in 2.4, 2.5, 3.1-3.4 & 4.1.

As with Test 1, you will not be expected to produce long, original proofs of challenging, never before seen propositions. I may, however, ask you to give a short proof or two of propositions that are new to you but which I consider basic, or a proof of one of the basic yet important results from class (see Theorems and Proofs section below). You may also be asked to prove a result (or variation thereof) taken from the homework problems, or I may ask you to explain some aspect of one of the longer proofs we worked through in class. In addition to the homework material, you should be familiar with the material outlined below.

## **Definitions and Concepts**

Be able to

- 1. State the definition of a Cauchy sequence (Definition 2.4.1).
- 2. State the definition of a series (Definition 2.5.1) and what is means for the series to converge.
- 3. State the definition of absolute convergence (Definition 2.5.12).
- 4. State the definition of the limit of a function (Definition 3.1.3).
- 5. Use the  $\epsilon \delta$  definition of a limit to prove particular limit results: see example 3.1.5.
- 6. Use sequential limits (Lemma 3.1.7) to prove limit results (Corollary 3.1.12 for example).
- 7. State the definition of a continuous function (Definition 3.2.1).
- 8. As with limits, use both the  $\epsilon \delta$  definition of continuity and sequential limits to prove continuity (or discontinuity) of functions. (See Examples 3.2.3 and 3.2.11).
- 9. State the definition of uniform continuity (Definition 3.4.1).
- 10. Apply the definition of uniform continuity to show that a particular function is uniformly continuous (see Example 3.4.3 and textbook Exercise 3.4.11.).
- 11. State the definition of the derivative (Definition 4.1.1).
- 12. Apply the definition of the derivative (Exercise 4.1.5)

## **Theorems and Proofs**

Know how to prove the following results:

1. Proposition 2.4.4: A Cauchy sequence is bounded.

- 2. Proposition 2.5.5 (not done in class, but a nice short test question)
- 3. Proposition 2.5.8:  $\sum x_n$  converges implies  $x_n \longrightarrow 0$ .
- 4. Proposition 2.5.13: absolute convergence of a series implies convergence.
- 5. Propostion 3.1.4: The limit of a function is unique (not done in class, but another nice, short test question.)
- 6. Proposition 3.2.7: compositions of continuous functions are continuous.
- 7. Lemma 3.3.1: A continuous function on a closed and bounded interval is bounded.
- 8. Proposition 4.1.4: Differentiability implies continuity.