For Test 1 you should be familiar with all homework problems assigned in Asn 1, 2, 3 & 4 as well as the theory covered up to and including Section 2.3 of the text.

You will not be expected to produce long, original proofs of challenging, never before seen propositions. I may, however, ask you to give a short proof or two of propositions that are new to you but which I consider basic and make use of the standard techniques we've seen (for example, showing two sets are equal by showing that each is a subset of the other, or using the definition of convergence of a sequence to prove that a given sequence converges to a particular limit.) You may also be asked to prove a result (or variation thereof) taken from the homework problems, or I may ask about some aspect of one of the proofs we worked through in class. In addition to the homework material, you should be familiar with the material outlined below.

Definitions and Concepts

Be able to

1. State the well ordering property of $\mathbb{N}$.
2. Give precise definitions of the domain and range of a function.
3. Define what it means for a function to be injective, surjective, bijective.
4. Define (in reference to sets) cardinality, finite, countably infinite, countable, uncountable.
5. Define (in reference to sets) bounded above/below, least upper bound, greatest lower bound.
6. Define ordered set, and state what it means for an ordered set to have the least upper bound property.
7. State what a bounded function is.
8. State the Archimedean property of the real numbers.
9. State the definition of the limit of a sequence $\lim_{n\to\infty} x_n$ and explain what it means for a sequence to converge.
10. State the definition of a monotone sequence.
11. State the definition of a subsequence.
12. State the definitions of $\limsup_{n\to\infty} x_n$ and $\liminf_{n\to\infty} x_n$.
13. For a particular sequence $\{x_n\}_{n=1}^{\infty}$, determine $\lim_{n\to\infty} x_n$ and prove your result using the ϵ, M definition.
14. Determine, with explanation, the $\limsup$ and $\liminf$ of a given sequence.

Theorems and Proofs

Know how to prove the following results:

1. Exercise 1.1.2.
2. Proposition 1.2.8.
3. Exercise 1.2.1
4. Exercise 1.4.1
5. A convergent sequence has a unique limit (Prop. 2.1.6).
6. A convergent sequence is bounded (Prop. 2.1.7)
7. Proposition 2.1.17
8. Exercises 2.1.1 to 2.1.7 are nice test-style exercises
9. Proposition 2.2.5 (i). We did 2.2.5.(iii) in class, but (i) is easier and more direct.
10. Exercise 2.3.5