Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

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1. For this question use the $\epsilon\delta$ definition of the limit:

   (a) Determine the limit (with proof) or show that it does not exist: $\lim_{x\to 1} \frac{1}{x}$.

   (b) Determine the limit (with proof) or show that it does not exist: $\lim_{x\to 9} \sqrt{x}$.

   (c) Determine the limit (with proof) or show that it does not exist: $\lim_{x\to c}(x^2+x+1)$ where c is any real number.

2. (Squeeze Law revisited) Let $S \subset \mathbb{R}$ and c be a cluster point of S. Suppose $f : S \to \mathbb{R}$, $g : S \to \mathbb{R}$ and $h : S \to \mathbb{R}$ are such that
$$f(x) \le g(x) \le h(x)$$
for every $x \in S$, and that
$$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L \in \mathbb{R} .$$
Prove that $\lim_{x\to c} g(x) = L$.

3. Let $S \subset \mathbb{R}$, c be a cluster point of S, and suppose that $f : S \to \mathbb{R}$ is bounded. Show that there exists a sequence $\{x_n\}_{n=1}^{\infty}$ such that (i) each $x_n \in S \setminus \{c\}$, (ii) $x_n \to c$, and (iii) the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges.

4. Prove that $f : \mathbb{R} \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ x^2 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$
is continuous at 1 but discontinuous at 2.

5. Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both continuous and that $f(r) = g(r)$ for every $r \in \mathbb{Q}$. Prove that $f(x) = g(x)$ for every $x \in \mathbb{R}$.