Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

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1. Prove that if $\sum_{k=1}^{\infty} x_k$ converges then $\sum_{k=1}^{\infty}(x_{2k} + x_{2k+1})$ also converges.

2. Suppose $\sum_{k=1}^{\infty} x_k$ is conditionally convergent, and define

$$a_k = \max\{x_k, 0\}$$
$$b_k = \min\{x_k, 0\}$$

   Show that $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ are both divergent.
   (Hint: $x_k = a_k + b_k$, Proposition 2.5.10 and contradiction).

3. Textbook exercise 2.5.3.

4. Prove that if $\sum_{k=1}^{\infty} x_k$ and $\sum_{k=1}^{\infty} y_k$ converge absolutely then so does $\sum_{k=1}^{\infty} x_k y_k$. Give an example where the converse is false.

5. If $\sum_{k=1}^{\infty} x_k$ converges absolutely prove that

$$\left|\sum_{k=1}^{\infty} x_k\right| \leq \sum_{k=1}^{\infty} |x_k| .$$

   (Hint: remember that series are limits of partial sums, and think about the triangle inequality.)