Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

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1. Show $\lim\limits_{n\to\infty} x_n = 0$ if and only if $\lim\limits_{n\to\infty} |x_n| = 0$.
2. Show that for every $x \in \mathbb{R}$, $\lim\limits_{n\to\infty} \dfrac{x^n}{n!} = 0$.
3. Show that $\lim\limits_{n\to\infty} \dfrac{n^2}{2^n} = 0$.
4. Textbook exercise 2.3.7.
5. Suppose $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are sequences and that $\lim_{n\to\infty} y_n = 0$. Further suppose that for each $k \in \mathbb{N}$, for any $m \geq k$ we have $|x_m - x_k| \leq y_k$. Prove that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence.
6. (a) Prove that for $r \neq 1$
$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$$
   (b) Prove that for $-1 < r < 1$
$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$
7. For $n \in \mathbb{N}$ let $s_n = \displaystyle\sum_{k=1}^{n} \frac{1}{k}$. Show that $s_{2^n} \geq 1 + n(1/2)$ for every $n \in \mathbb{N}$.