Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

1. Suppose that $S \subset \mathbb{R}$ and that S is nonempty and bounded. Show that for every $\epsilon > 0$ there is some $x \in S$ such that
$$\sup(S) - \epsilon < x \leq \sup(S)$$
(The proof is very short!)
2. Let F be an ordered field and $x, y \in F$. If $0 < x < y$ show that $x^2 < y^2$. (Use only the properties of ordered fields here: Definition 1.1.7 and Proposition 1.1.8 of the textbook.)
3. Let F be an ordered field and $x, y \in F$. Show that $x^2 + y^2 = 0$ if and only if $x = 0$ and $y = 0$. Again, use only properties of ordered fields here.
4. Let $x \geq 0$ be a real number. Show that there exists $n \in \mathbb{N}$ such that $n - 1 \leq x < n$.
5. Prove the arithmetic-geometric mean inequality: for x, y positive and real, $\sqrt{xy} \leq \dfrac{x+y}{2}$ with equality if and only if $x = y$.
6. Let $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ be bounded and nonempty. Let $C = \{a + b \mid a \in A, b \in B\}$. Show that $\sup(C) = \sup(A) + \sup(B)$.
7. Prove that for real numbers x and y, $\max\{x, y\} = \dfrac{x + y + |x - y|}{2}$.