Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

1. Show that if $|A| = n$ then $|\mathcal{P}(A)| = 2^n$.
2. Show that if B is countable and $A \subset B$ then A is countable.
3. Let $\mathbb{Q}[x]$ be the set of all polynomials in x with coefficients from $\mathbb{Q}[x]$. That is,
$$\mathbb{Q}[x] = \left\{ \sum_{k=0}^{n-1} a_k x^k \;:\; a_k \in \mathbb{Q} \text{ and } n \in \mathbb{N} \right\} ,$$
(here we define $x^0 = 1$) . Show that $\mathbb{Q}[x]$ is countable.
4. An element $a \in \mathbb{R}$ is called algebraic over $\mathbb{Q}$ if $p(a) = 0$ for some non-zero $p \in \mathbb{Q}[x]$. Show that the set of elements of $\mathbb{R}$ which are algebraic over $\mathbb{Q}$ is countably infinite. (Here you may assume the following corollary of the Fundamental Theorem of Algebra: if $p \in \mathbb{Q}[x]$ has degree n then p has at most n real roots.)
5. Show that if A is a finite nonempty subset of an ordered set then $\sup(A) \in A$.
6. Show that if A is a subset of an ordered set and $b \in A$ is an upper bound for A then $b = \sup(A)$.
7. Let S be an ordered set with the least upper bound property and A a nonempty subset that is bounded above. Show that if $\sup(A) \notin A$ then A is infinite.