Question 1:

(a)[3 points] Let $f(x) = \arcsin(\sinh(x))$. Evaluate $f'(0)$.

(b)[4 points] Evaluate

$$\lim_{x\to 0} \frac{e^{x^2} - 1}{x \sin x}$$

(c)[5 points] Evaluate

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$$

Question 2:

(a)[4 points] A particle is moving with acceleration $a(t) = t + \sin t$ m/s^2. If velocity at time $t = 0$ is $v(0) = 2$ m/s and initial position is $s(0) = 0$ m, determine $s(t)$, the position of the particle at time $t =$ seconds.

(b)[3 points] The average value of $f(x) = x^3$ over the interval $[0, a]$ is 16. Determine the value of a.

(c)[3 points] Use the Fundamental Theorem of Calculus to determine $f(x)$ if

$$\int_0^x f(t)\,dt = xe^{x^2} + 1 \ .$$

Question 3:

(a)[3 points] Evaluate:

$$\int \frac{2x^3 + 5\sqrt{x} - 3}{x}\, dx$$

(b)[3 points] Evaluate:

$$\int \frac{\sin\left(\sqrt{x} + 1\right)}{\sqrt{x}}\, dx$$

(c)[4 points] The population is growing at a rate of $P'(t) = 200e^{t/5}$ individuals per year, where year $t = 0$ corresponds to the present. If the current population size is $P(0) = 1000$, determine the population size at the end of five years.

Question 4:

(a)[6 points] Evaluate:

$$\int (x^2 + 1)e^{-x}\,dx$$

(b)[6 points] Evaluate:

$$\int \sec^6 x\ \tan^2 x\,dx$$

Question 5 [8 points]: Evaluate:

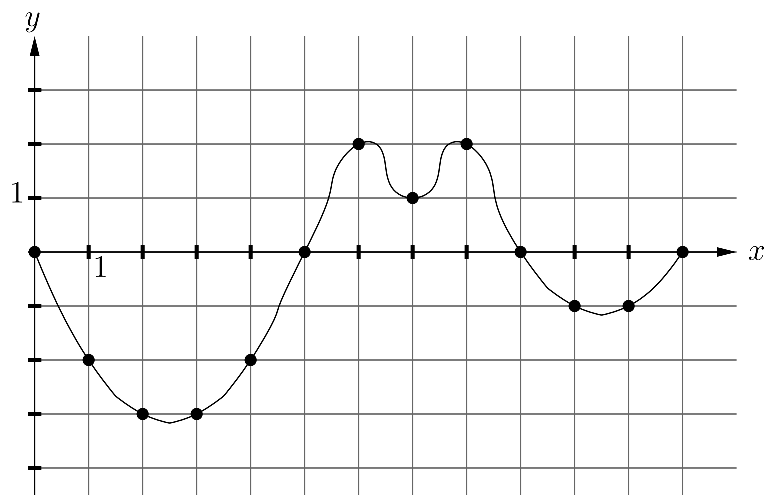
$$\int x^3\sqrt{4-x^2}\,dx$$

Question 6 [8 points]: Evaluate:

$$\int_0^1 \frac{4x^2 + x + 3}{(x+1)(x^2+1)}\,dx$$

Question 7:

(a)[**4 points**] Use the graph of $y = f(x)$ below to approximate $\int_0^{12} f(x)\,dx$ using M_6, the midpoint rule on six subintervals.



(b)[**4 points**] If Simpson's Rule is used to approximate $\int_0^{180} \left(\frac{x^4 + x}{24}\right) dx$, how many subintervals are required to guarantee an error of at most 1? Recall, the error in using Simpson's rule to approximate $\int_a^b f(x)\,dx$ is at most $\frac{K(b-a)^5}{180n^4}$, where $|f^{(4)}(x)| \le K$ on $[a, b]$.

Question 8:

(a)[3 points] Use the comparison theorem to determine if the integral converges or diverges (do not attempt to evaluate the integral):

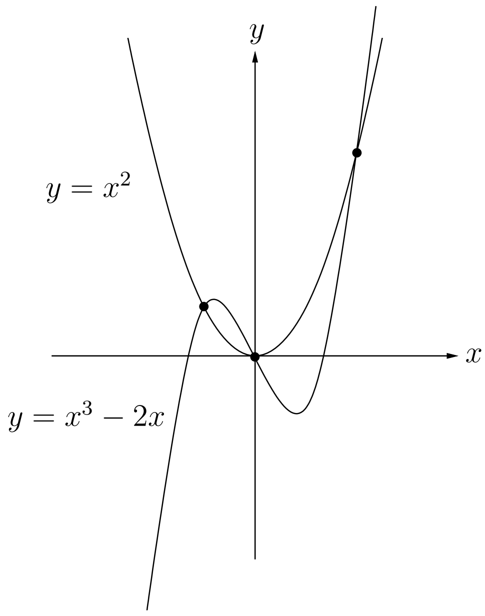
$$\int_1^\infty \frac{e^{-x}}{x^4+1}\,dx$$

(b)[5 points] Evaluate the improper integral:

$$\int_0^1 \frac{e^x}{\sqrt{e^x-1}}\,dx$$

Question 9:

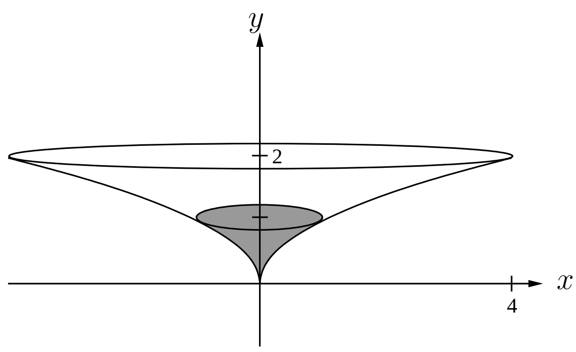
(a)[**5 points**] Determine the area of the region bounded between the curves $y = x^2$ and $y = x^3 - 2x$.



(b)[**5 points**] A 10 m chain with a 100 N weight at the end hangs over the side of a bridge. The weight is initially 20 m above the ground, and one metre of chain weighs 10 N. A person pulls the chain and weight up onto the bridge deck. How much work is done?

Question 10

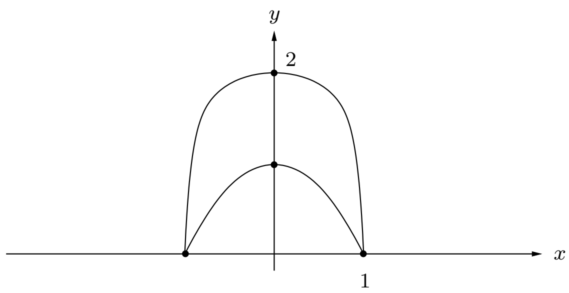
(a)[7 points] A vessel is formed by rotating the curve $y = \sqrt{x}$ about the y-axis as shown below. The vessel has a top radius of 4 m and a depth of 2 m. If the vessel is initially filled with water to a depth of 1 m, how much work is required empty the vessel by pumping the water up and over the top rim? Recall that the density of water is $\rho = 1000$ kg/m^3 and acceleration due to gravity is $g = 9.8$ m/s^2. (You may leave your final answer as a product of fractions.)



(b)[3 points] Determine the total volume of the vessel described in part (a). (Disks would be best here.)

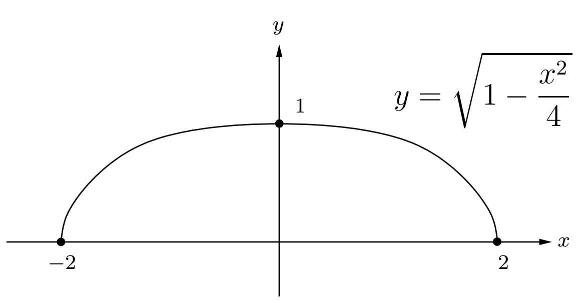
Question 11:

(a)[3 points] The region bounded between the curves $y = 1 - x^2$ and $y = 2(1 - x^4)$ is rotated about the line $x = -2$. Set up the integral representing the volume of the resulting solid. DO NOT EVALUATE THE INTEGRAL. (Cylindrical shells would be best here.)



(b)[3 points] Referring to the diagram for part (a), the same bounded region is rotated about the horizontal line $y = 2$. Set up the integral representing the volume of the resulting solid. DO NOT EVALUATE THE INTEGRAL. (Disks would be best here.)

(c)[4 points] The base (flat bottom surface) of the solid S is the region between the curve $y = \sqrt{1 - \frac{x^2}{4}}$ and the x-axis. Cross-sections perpendicular to the x-axis are squares. Determine the volume of S.



Question 12:

(a)[4 points] Solve the differential equation:

$$\frac{dy}{dx} = \frac{y\cos x}{1+y^2} \qquad y(0) = 1$$

(b)[6 points] A tank contains 1000 L of pure water. Salt water with a concentration of 0.1 kg/L of salt enters the tank at a rate of 10 L per minute, while water is pumped out of the bottom of the tank at the same rate. The water in the tank is kept thoroughly mixed. The mass $A(t)$ of dissolved salt in the tank at time t is modeled by the differential equation

$$\frac{dA}{dt} = 1 - \frac{1}{100}A \qquad A(0) = 0$$

Solve this differential equation for $A(t)$.

Question 13:

(a) [**4 points**] Use the definition to construct $T_2(x)$, the Maclaurin series of degree 2, for the function $f(x) = \sin(x^2 + x)$

(b) [**4 points**] Use Maclaurin series (not L'Hospital's Rule) to evaluate the limit

$$\lim_{x \to 0} \frac{\cos x - 1}{x \sin x}$$