Question 1:

(a) Determine the coefficient of $1/(z+1)$ in the partial fraction decomposition of

$$\frac{z^3+4z+9}{(2z+2)(z-3)^5}$$

[3]

(b) Compute $\displaystyle\int_\Gamma \frac{z^3+4z+9}{(2z+2)(z-3)^5}\,dz$ where Γ is the circle of radius 3 centred at $z=-1$. Explain your reasoning.

[2]

Question 2: Show that $\overline{\sin(z)} = \sin(\overline{z})$.

[5]

Question 3: Find all solutions to $e^{2z} + e^z + 1 = 0$

[5]

Question 4: Compute $(1+i)^{(1+i)}$

[5]

Question 5: Let Γ be the contour consisting of two smooth curves γ_1 and γ_2 as follows: γ_1 is the straight line segment from i to $2i$. γ_2 is the arc of the circle of centre $z = 0$ and radius 2 which begins at $2i$ and proceeds clockwise, ending at $\sqrt{2}(1 + i)$. Using this contour evaluate

$$\int_\Gamma \frac{1}{(\overline{z})}\, dz$$

[6]

Question 6: Using the same contour Γ as in the previous question, evaluate

$$\int_\Gamma \frac{1}{z}\, dz$$

[4]

Question 7: Compute the following integrals. In each case the contour is traversed once in the positive direction. Make reference to any theorems used.

(a) $\displaystyle\int_\Gamma \frac{\sin(z)}{z^2 - z - 6}\, dz$

Here Γ is the circle of centre $z = i$ and radius 2.

[3]

(b) $\displaystyle\int_\Gamma \frac{e^z}{z^2 + 1}\, dz$

Here Γ is the top half of the circle of centre $z = 0$ and radius 2 followed by the line segment from -2 to 2.

[3]

(c) $\displaystyle\int_\Gamma \frac{\cos^2(z)}{z^2 - \pi^2}\, dz$

Here Γ is a square of area 64 and is such that each side is parallel to one coordinate axis and bisected by another.

[4]