

Question 1: Simplify and express your answer in the form $a + ib$ where a and b are real:

$$\left[1 + \frac{3}{(1 + 2i)}\right]^2$$

[5]

Question 2: Express $z = \frac{i(1 - i)(-\sqrt{3} + i)}{5}$ in form $z = re^{i\theta}$ where r and θ are real.

[5]

Question 3: Determine all values of $(-32 - 32i)^{1/5}$.

[5]

Question 4: Find all solutions to $(z + 1)^5 = z^5$. (Hint: think about $\omega = 1^{1/5}$. You may leave your answer in terms of ω .)

[5]

Question 5: Let $S = \{z \in \mathbb{C} \mid 1 \leq |z| \leq 2 \text{ and } 0 \leq \text{Arg}(z) \leq \pi/4\}$ and $f(z) = z^2 + i$. Sketch S and $f(S)$, the image of S under the mapping f .

[5]

Question 6: Determine all points where the function $f(z) = x^2 - y^2 + 2x + 1 + 2ixy + i2y$ is analytic. Include all details in your conclusion.

[5]

Question 7:

(a) Show that $u(x, y) = -e^{-x} \sin(y) + x$ is harmonic on \mathbb{R}^2 .

[3]

(b) Find a harmonic conjugate $v(x, y)$ for $u(x, y)$ given in part (a)

[5]

(c) Express the analytic function $f(z) = u(x, y) + iv(x, y)$ as a function of z only.

[2]