1. Compute the following integrals. Explain your reasoning, especially if relying on the path independence theorem.

   (a) $\int_\gamma 2z\,dz$ where γ is parametrized by $z(t) = 2\cos^3(\pi t) - i\sin^2(\pi t/4), 0 \le t \le 2$.

   (b) $\int_\gamma \frac{1}{z}\,dz$ where γ is the right half of the unit circle from $-i$ to i .

   (c) $\int_\gamma \frac{1}{z}\,dz$ where γ is the left half of the unit circle from $-i$ to i .

   (d) $\int_\gamma z\sin(z^2)\,dz$ where γ is the spiral $z(t) = te^{it}, 0 \le t \le 8\pi$.

   (e) $\int_\gamma \mathrm{Log}(z)\,dz$ where γ is the unit circle . (Harder: be careful with the branch cut; you may require limits.)

2. Compute $\int_\Gamma \left[\frac{7}{(z-i)^3} - \frac{5}{z-i} + 11(z-i)^5)\right] dz$ where Γ is the circle $|z-i| = 3$ traversed once in the positive direction.

3. Proof or counterexample: If f is analytic at each point of a closed contour Γ then $\int_\Gamma f(z)\,dz = 0$.

4. Let D be the unit disk $\{z : |z| < 1\}$ and suppose that f is analytic with $|f'(z)| < M$ for every $z \in D$. Show that
$$|f(z_2) - f(z_1)| \le M|z_2 - z_1|$$
for every $z_1, z_2 \in D$.

   (Hint: $f(z_2) - f(z_1) = \int_\Gamma f'(z)\,dz$ where Γ is a line segment from z_1 to z_2 in D.)