1. Show that $u(x, y) = \ln(x^2 + y^2)$ is harmonic in an appropriate domain D and then find a harmonic conjugate $v(x, y)$ for $u(x, y)$.

2. (a) Show that $v(x, y) = \dfrac{x}{x^2 + y^2}$ is harmonic on $D = \mathbb{C} \setminus \{0\}$.

    (b) Find a function $f(z) = u(x, y) + iv(x, y)$ that is analytic on D .

    (c) Express $f(z)$ in (b) in terms of the variable z.

3. Suppose $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ is a polynomial of degree $n \geq 1$ and that $|a_0| > 1$. Show that $p(z)$ has at least one zero outside the unit circle.

4. Find the Taylor form for $p(z) = (z - 1)(z - 2)^3$ centred at $z = 2$.

5. Find all poles and their corresponding multiplicities for $f(z) = \dfrac{z^2 - 9}{(z^2 + 9)^2}$, and then determine the coefficient of $\dfrac{1}{z - 3i}$ in the partial fraction decomposition of $f(z)$. (That coefficient is called the **residue** of $f(z)$ at $3i$, written Res(3i).)

6. (a) Calculate $\tan(\pi - 2i)$
    (b) Find all solutions to $\cos(z) = i\sin(z)$
    (c) Find all solutions to $\cos(z) = \sin(z)$

7. Compute the following logarithms:

    (a) $\log(\sqrt{2} + \sqrt{6}i)$
    (b) $\log(-ei)$
    (c) $\text{Log}(-e^2)$
    (d) $\text{Log}(-12 + 5i)$
    (e) $\text{Log}[(1 + \sqrt{3}i)^5]$

8. Find the image of the given set under the mapping $w = \text{Log}z$:

    (a) The circle $|z| = 4$
    (b) the region in the first quadrant bounded between the circles $|z| = 1$ and $|z| = e$

9. Prove that $\log(z^n) = n\log(z)$ for every $z \in \mathbb{C} \setminus \{0\}$ and integer n.

10. Find all values of the given complex power:

    (a) $(1 + i)^{1-i}$
    (b) $3^{2i/\pi}$
    (c) $(1 + \sqrt{3}i)^i$

11. Find the principal value of the given complex power:

    (a) $i^{i/\pi}$
    (b) $(1 + i)^{2-i}$

12. Is $1^z = 1$ for every complex number z? Is the principal value of $1^z = 1$? Explain.