NOTE: Some of the exercises require that you sketch a set. When doing so, clearly indicate which points are in the set and which points are not. For example, boundary lines or curves which are not in the set should be indicated with dashes or dots.

1. Express each of the following functions in the form $w = u(x, y) + iv(x, y)$ where u and v are real:

   (a) $f(z) = \dfrac{\overline{z}}{z+1}$

   (b) $f(z) = z + \dfrac{1}{z}$

   (c) $f(z) = e^{2z+i}$

   (d) $f(z) = e^{z^2}$

2. Find the domain of definition of each of the following:

   (a) $f(z) = 2\text{Re}(z) - iz^2$

   (b) $f(z) = \dfrac{iz}{|z| - 1}$

   (c) $f(z) = \dfrac{3z + 2i}{z^3 + 4z^2 + z}$

   (d) $f(z) = e^{1/(e^z - 1)}$

3. For each of the following, sketch the set S and then sketch the image of S under the given function:

   (a) $S = \{z \in \mathbb{C} \mid \text{Re}(z) > 0 \text{ and } \text{Im}(z) > 0\}\,, \quad f(z) = -3iz$

   (b) $S = \{z \in \mathbb{C} \mid \text{Re}(z) > 1\}\,, \quad f(z) = (z+2)^2$

   (c) $S = \left\{z \in \mathbb{C} \mid |\text{Arg(z)}| < \dfrac{\pi}{12}\right\}, \quad f(z) = z^3$

   (d) $S = \left\{z \in \mathbb{C} \mid \dfrac{\pi}{4} < \text{Im}(z) < \dfrac{\pi}{2}\right\}, \quad f(z) = e^z$

4. Prove that $\overline{(e^z)} = e^{\overline{z}}$.

5. Find the following limits (if the limit does not exist, say so):

   (a) $\lim\limits_{z \to i} 3z^2 + 7iz - 2 - i$

   (b) $\lim\limits_{z \to i} \dfrac{z^4 + 1}{z - i}$

   (c) $\lim\limits_{z \to i} \dfrac{z^4 - 1}{z - i}$

   (d) $\lim\limits_{z \to 1+i} \dfrac{z^2 + z - 1 - 3i}{z^2 - 2z + 2}$

6. Give an argument which shows that $\lim\limits_{z \to 0} \dfrac{\text{Re}(z)}{\text{Im}(z)}$ does not exist.

7. Determine where each of the following is continuous. State your answer as a set:

   (a) $f(z) = \dfrac{z+2}{z^2+4}$

   (b) $f(z) = \dfrac{z}{e^z+1}$

   (c) $f(z) = \dfrac{z}{|e^z|-1}$

8. Determine the set of points on which $f(z) = \left(\dfrac{(2-i)z+1}{z^2+9i}\right)^3$ is analytic. (Don't use the Cauchy-Riemann equations for this. Rather, use Theorem 3 in 2.3).

9. For each of the following determine the set of points on which the function is differentiable:

   (a) $f(z) = e^{x^2-y^2}\cos(2xy) + ie^{x^2-y^2}\sin(2xy)$

   (b) $f(z) = \dfrac{x^3+xy^2+x}{x^2+y^2} + i\dfrac{x^2y+y^3-y}{x^2+y^2}$

10. Find real constants a, b, c and d so that $f(z) = x^2+axy+by^2+i(cx^2+dxy+y^2)$ is entire.

11. The function $f(z) = x^2-x+y+i(y^2-5y-x)$ is not analytic at any point, but is differentiable along a line in the complex plane. Find the line.