1. Express each of the following complex numbers in the form $a + ib$ with a and b real:

   (a) $2i^6 + \left(\frac{2}{-i}\right)^3 + 5i^{-5} - 12i$

   (b) $i(4 - i) + 4i(1 + 2i)$

   (c) $3i + \frac{1}{2 - i}$

   (d) $\frac{(3 - i)(2 + 3i)}{1 + i}$

   (e) $(2 + 3i)\left(\frac{2 - i}{1 + 2i}\right)^2$

2. Find both $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ if $z = \left(\frac{i}{3 - i}\right)\left(\frac{1}{2 + 3i}\right)$.

3. Let $z = x + iy$. Express $\mathrm{Im}(2z + 4\overline{z} - 4i)$ in terms of x and y .

4. Find all solutions z to the following equations:

   (a) $z - 2\overline{z} + 7 - 6i = 0$

   (b) $z = \overline{z}$

   (c) $z^2 = (\overline{z})^2$

5. Show that if $z_1 z_2$ is a nonzero real constant then $z_2 = k\overline{z_1}$ for some real number k .

6. Let $z_1 = 5 + 4i$ and $z_2 = -3i$. Sketch the following vectors in the complex plane:

   (a) $3z_1 + 5z_2$

   (b) $z_1 - 2z_2$

7. Let $z_1 = 10 + 8i$ and $z_2 = 11 - 6i$. Which of these complex numbers is closest to $1 + i$?

8. Sketch and describe the set of points in the complex plane which satisfy the following:

   (a) $|z - 1| = 1$

   (b) $|z - 2| = \mathrm{Re}(z)$

9. Write the following complex numbers in polar form using $\theta = \mathrm{Arg}(z)$:

   (a) $6i$

   (b) $\frac{3}{-1 + i}$

10. Express $(3 - 3i)(5 + 5\sqrt{3}i)$ in polar form. Your final answer should not contain any inverse trigonometric functions.

11. Express $(1 + \sqrt{3}i)^9$ in both polar form and form $a + ib$.