Math 372 - Introductory Complex Variables

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Jan 16 2017

1.6 - Planar Sets

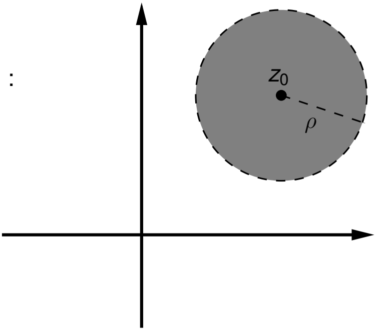
Open Disks

Definition: Let $z_0 \in \mathbb{C}$ and $\rho > 0$ be real. The set

$$\{z : |z - z_0| < \rho\}$$

is called an open disk of radius ρ and centre z_0. The set is sometimes called a circular neighbourhood of z_0.

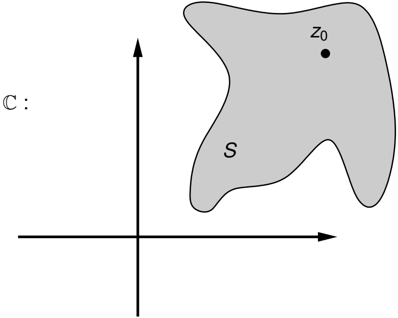
$\mathbb{C}$:



Interior Point

Definition: Let $S \subset \mathbb{C}$ be a set and $z_0 \in S$. z_0 is an interior point of S if there is some circular neighbourhood of z_0 completely contained in S. That is, there is some $\rho > 0$ such that

$$\{z : |z - z_0| < \rho\} \subset S$$



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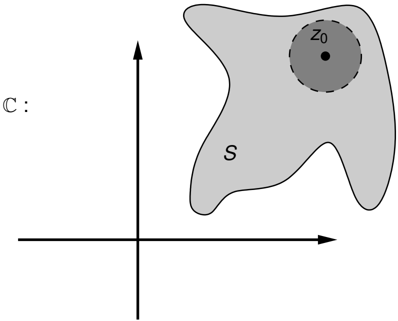

Open Set

Definition: A set $S \subset \mathbb{C}$ if open if every point of S is an interior point.

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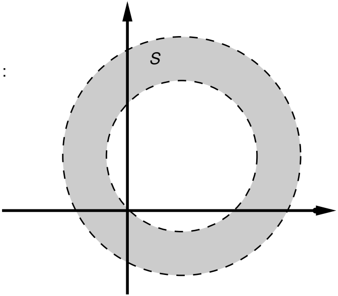

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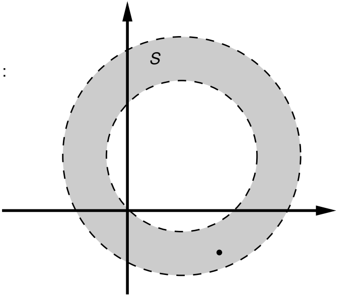

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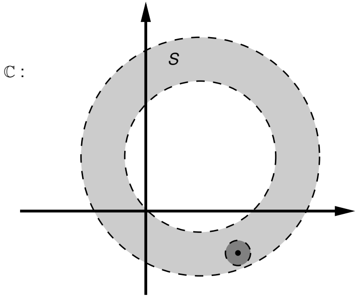
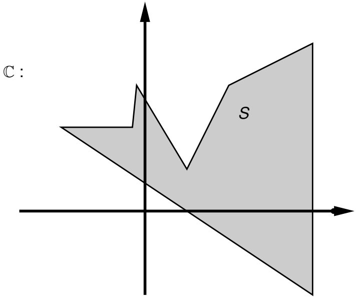


S is open



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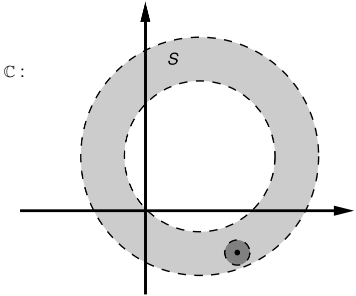

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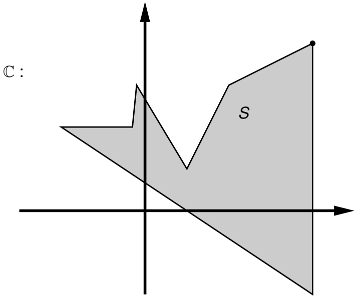

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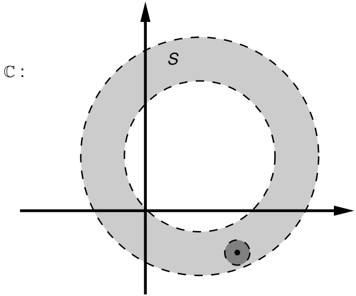

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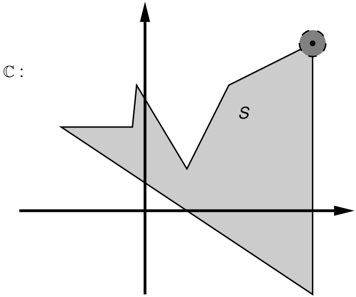

S is not open

Connected Set

Definition: An open set $S \subset \mathbb{C}$ if connected if any two points S can be joined by a path consisting of a finite number of line segments which lie entirely in S.

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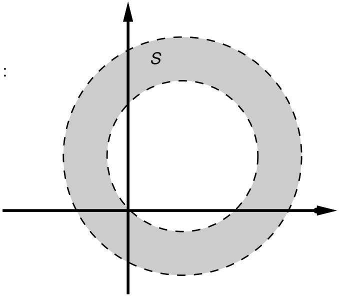

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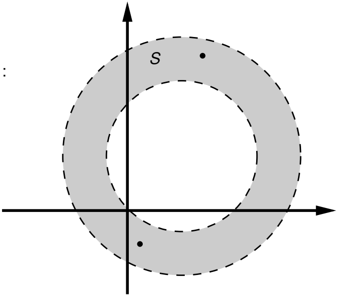

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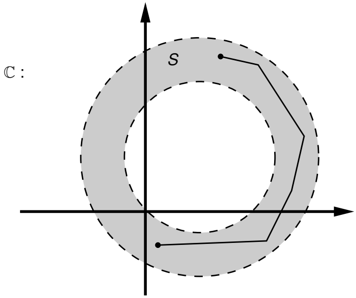
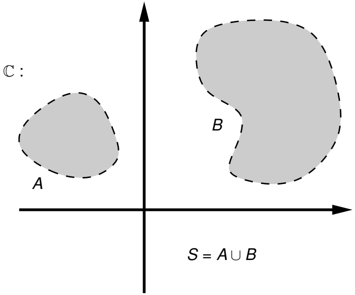


S is connected



A and B are each connected, but $S = A \cup B$ is not

Domain, Boundary Point, Closed sets

Definition: A domain is an open connected set.

Definition: Let $S \subset \mathbb{C}$. z_0 is called a boundary point of S if every circular neighbourhood of z_0 contains at least one point in S and one point not in S.

Definition: A set $S \subset \mathbb{C}$ is called closed if it contains all of its boundary points.

Bounded sets, Compact sets, Regions

Definition: A set $S \subset \mathbb{C}$ is bounded if there is $R \in \mathbb{R}$ such that $|z| < R$ for every $z \in S$.

Definition: A $S \subset \mathbb{C}$ is compact if it is both closed and bounded.

Definition: A region is a domain together with some, none, or all of its boundary points.

Example

Let S be the set of complex numbers which satisfy
$1 < (\mathrm{Im}(z))^2 < 4$.

1. Is S open?

2. Is S connected?

3. Is S a domain?

4. Is S bounded?

5. Describe the boundary points.

6. Is S a region?