1. Textbook Exercise 5.1
2. Textbook Exercise 5.8
3. Textbook Exercise 5.9
4. This exercise covers some of the equivalent methods of demonstrating bijectivity that we mentioned in class. The text does not formally define bijection, but the definition is implicit in the definition of homeomorphism on page 56. Extracting that implied definition:

   Definition: For sets X and Y, $f : X \to Y$ is a ***bijection*** if there exists $g : Y \to X$ such that $(g \circ f)(x) = x$ for each $x \in X$ and $(f \circ g)(y) = y$ for each $y \in Y$.

   (i) Give an example of topological spaces X and Y and functions f and g such that $(g \circ f)(x) = x$ for each $x \in X$, but it is not the case that $(f \circ g)(y) = y$ for each $y \in Y$.

   (ii) Show that $f : X \to Y$ is bijective if it is both surjective and injective.

   (iii) Show that for topological spaces X and Y, $f : X \to Y$ is bijective if and only if the equation $f(x) = y$ has exactly one solution x for each $y \in Y$.

5. We saw that for a disjoint union $(X \sqcup Y, \mathcal{T}_{X \sqcup Y})$, open sets have the form $U \sqcup V$ where $U \in \mathcal{T}_X$ and $V \in \mathcal{T}_Y$. Find the general form of a closed set in $X \sqcup Y$.
6. Let $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ be a permutation and define
$$f : \mathbb{R}^n \to \mathbb{R}^n : (x_1, x_2, \ldots, x_n) \mapsto \left(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)}\right)$$
   Show that f is continuous.
7. (General product space topology) Let $\{X_\alpha \mid \alpha \in \mathcal{A}\}$ be an indexed set of topological spaces and define
$$X = \prod_{\alpha \in \mathcal{A}} X_\alpha \ .$$
   For $x \in X$, x has "coordinates" x_α where α ranges over $\mathcal{A}$, and we write $x = (x_\alpha)_{\alpha \in \mathcal{A}}$. In other words, each x is a function of $\alpha \in \mathcal{A}$. For example, if $\mathcal{A} = \mathbb{N}$ and each $X_\alpha = \mathbb{R}$, then
$$X = \prod_{\alpha \in \mathcal{A}} X_\alpha = \prod_{\alpha \in \mathbb{N}} \mathbb{R} = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \ldots$$
   and for $x \in X$, $x = (x_1, x_2, x_3, \ldots)$ where each $x_i \in \mathbb{R}$.

   The open sets of $X = \prod_{\alpha \in \mathcal{A}} X_\alpha$ are defined to be unions of sets of the form $\prod_{\alpha \in \mathcal{A}} U_\alpha$ where each U_α is open in X_α and $U_\alpha = X_\alpha$ for all but finitely many α (notice this last condition: important!).

   Show that $X = \prod_{\alpha \in \mathcal{A}} X_\alpha$ with this definition of open set is indeed a topological space.