Note: Structure your proofs using the solutions to previous assignments as models. State the full "**Proposition:**" followed by "**Proof:**", indent your argument where appropriate for readability and be careful with notation.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

1. Textbook Exercise 4.5
2. Textbook Exercise 4.6
3. Textbook Exercise 4.7
4. Textbook Exercise 4.8
5. Textbook Exercise 4.9
6. Let $(X, \mathcal{T}_X)$ be a topological space. Prove that $S \subset X$ is open if and only if for each $x \in S$ there is $U \in \mathcal{T}_X$ such that $x \in U \subset S$.
7. Proof or counterexample: if $(X, \mathcal{T}_X)$ is a compact topological space and $S \subset X$ has the induced topology then S is compact.
8. For $n \in \mathbb{N}$ define an ***open cell*** in $\mathbb{R}^n$ to be the Cartesian product of n open intervals from $\mathbb{R}$:

   $$(a_1, b_1) \times (a_2, b_2) \times \cdots \times (a_n, b_n)$$

   Show that the collection of all open cells in $\mathbb{R}^n$ is a basis for $\mathbb{R}^n$.