Question 1:

(a) Find the linear approximation $T_1(x)$ about $a = 1$ for $f(x) = \sqrt{x+3}$.

[5]

(b) Suppose you use your result in part (a) to approximate $\sqrt{4.1}$. Give an error bound on the resulting approximation.
Note: There is no need to actually calculate the approximation to $\sqrt{4.1}$, you are only being asked to determine a bound on the associated error. Express your answer as a single simplified fraction.

[5]

Question 2:

(a) Find $T_3(x)$, the Taylor polynomial of degree 3, for $f(x) = x\ln(x) - x$ about $a = 1$.

[5]

(b) Give an error bound if $T_3(x)$ in part (a) is used to approximate $f(1/2)$.
Note: Again with this question: there is no need to actually calculate the approximation to $f(1/2)$, you are only being asked to determine a bound on the associated error. Express your answer as a single simplified fraction.

[5]

Question 3:

(a) Find the first three nonzero terms of the Maclaurin series for $f(x) = e^{(x^2)} \cos(3x)$.

[6]

(b) The degree 6 term of the Maclaurin series for $f(x) = e^{(x^2)} \cos(3x)$ is $\dfrac{67x^6}{240}$. Determine $f^{(6)}(0)$, the sixth derivative of f at 0. Simplify your final answer.

[4]

Question 4: The Maclaurin series for $e^{\arctan(x)}$ is

$$e^{\arctan(x)} = 1 + x + \frac{x^2}{2} - \frac{x^3}{6} - \frac{7x^4}{24} + \cdots$$

Use this to find the first three nonzero terms of the Maclaurin series for $g(x) = \dfrac{e^{\arctan(x)}}{1+x^2}$. (There are several ways to do this, but one way is much easier than the others.)

[4]

Question 5: Evaluate the following limit:

$$\lim_{x \to 0} \frac{x^5 \sin(x) - e^{(x^6)} + 1}{x^8}$$

[6]

Question 6: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k k! x^{2k}}{3^k}$$

[5]

Question 7: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{(x+2)^k}{(k+1)^3 2^k}$$

[5]