1. Find an example of a bounded discontinuous function $f : [0,1] \to \mathbb{R}$ that has neither an absolute maximum nor absolute minimum. Show that your function has the required properties.

2. Show that if $f : [a,b] \to \mathbb{R}$ is continuous then $f([a,b])$ is either a closed and bounded interval or a single real number.

3. Show that if $f : (0,1) \to \mathbb{R}$ is bounded and continuous then $g(x) = x(1-x)f(x)$ is uniformly continuous on $(0,1)$.

4. Show that $f : (0,\infty) \to \mathbb{R}$ defined by $f(x) = \sin(1/x)$ is not uniformly continuous.

5. (Product Rule) Suppose $f : (a,b) \to \mathbb{R}$ and $g : (a,b) \to \mathbb{R}$ are both differentiable at $c \in (a,b)$. Show that $h = fg$ is differentiable at c and that

$$h'(c) = f(c)g'(c) + f'(c)g(c)$$

6. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$

Show that f is differentiable at 0.