1. (a) Determine the limit (with proof) or show that it does not exist: $\lim_{x\to c} \sqrt{x}$ where $c \geq 0$.

   (b) Determine the limit (with proof) or show that it does not exist: $\lim_{x\to c}(x^2 + x + 1)$ where c is any real number.

2. (Squeeze Law revisited) Let $S \subset \mathbb{R}$ and c be a cluster point of S. Suppose $f : S \to \mathbb{R}$, $g : S \to \mathbb{R}$ and $h : S \to \mathbb{R}$ are such that
$$f(x) \leq g(x) \leq h(x)$$
for every $x \in S$, and that
$$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L \in \mathbb{R} .$$
Prove that $\lim_{x\to c} g(x) = L$.

3. Let $S \subset \mathbb{R}$, c be a cluster point of S, and suppose that $f : S \to \mathbb{R}$ is bounded. Show that there exists a sequence $\{x_n\}_{n=1}^{\infty}$ such that each $x_n \in S \setminus c$, $x_n \to c$, and $\{f(x_n)\}_{n=1}^{\infty}$ converges.

4. Prove that $f : (0, \infty) \to \mathbb{R} : x \mapsto 1/x$ is continuous using the definition of continuity.

5. Prove that $f : \mathbb{R} \to \mathbb{R}$ defined by
$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ x^2 & \text{if } x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$$
is continuous at 1 but discontinuous at 2.

6. Suppose $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both continuous and that $f(r) = g(r)$ for every $r \in \mathbb{Q}$. Prove that $f(x) = g(x)$ for every $x \in \mathbb{R}$.

7. Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and that $f(c) > 0$. Show that there is some $\alpha > 0$ such that $f(x) > 0$ for every $x \in (c - \alpha, c + \alpha)$.