1. Suppose $\{x_n\}_{n=1}^{\infty}$ is bounded. Show that $\lim_{n\to\infty} x_n = x$ if and only if every subsequence converges to x.

2. **Definition:** $x \in \mathbb{R}$ is called a **cluster point** of $S \subset \mathbb{R}$ if for every $\epsilon > 0$, $(x-\epsilon, x+\epsilon) \cap S \setminus x \neq \emptyset$.

   Prove that if $S \subset \mathbb{R}$ is bounded and infinite then there is at least one cluster point of S. (see exercise 2.3.9 of text for a hint.)

3. Suppose $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are sequences and that $\lim_{n\to\infty} y_n = 0$. Further suppose that for each $k \in \mathbb{N}$, for any $m \geq k$ we have $|x_m - x_k| \leq y_k$. Prove that $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

4. (a) Prove that for $r \neq 1$
$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$$
   (b) Prove that for $-1 < r < 1$
$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

5. Prove that if $\displaystyle\sum_{k=1}^{\infty} x_k$ converges then $\displaystyle\sum_{k=1}^{\infty}(x_{2k} + x_{2k+1})$ also converges.

6. Suppose $\displaystyle\sum_{n=1}^{\infty} x_n$ is conditionally convergent, and define
$$a_n = \max\{x_n, 0\}$$
$$b_n = \min\{x_n, 0\}$$
   Show that $\displaystyle\sum_{n=1}^{\infty} a_n$ and $\displaystyle\sum_{n=1}^{\infty} b_n$ are both divergent.

7. (A bit more challenging) Find an infinite collection of closed discs $D_1, D_2, D_3, \ldots$ in the plane with centres $c_1, c_2, c_3, \ldots$, respectively, such that
   (a) Every line in the plane intersects at least one of the D_i, and
   (b) The sum of the areas of the D_i is finite.

   (hint: Let $c_n = \sum_{k=1}^{n} 1/k$. Think about covering the positive x-axis with disks having centres $(c_n, 0)$ and radii which ensure the disks overlap. Then apply this same construction to the remaining sections of the axes.)