1. Prove that $|\mathbb{N}| = |\mathbb{Z}|$ by finding an explicit bijection between the two sets and showing that your function is indeed a bijection.
2. Suppose $A \subset B \subset C$ and $|B| \leq |C|$. Show that $|A| \leq |C|$. (This is a very short proof, and as was pointed out in class, the statement that $|B| \leq |C|$ is unnecessary.)
3. Let $f : A \to B$ and $g : B \to C$. Prove
   (a) If $g \circ f$ is injective then f is injective.
   (b) If $g \circ f$ is surjective then g is surjective. (Note: the original statement of part (b) was incorrect. The original claim was that f is surjective if $g \circ f$ is; can you find a counterexample showing the claim is false?)
4. Let F be an ordered field and $x, y \in F$. If $0 < x < y$ show that $x^2 < y^2$. (Use only the properties of ordered fields here: Definition 1.1.7 and Proposition 1.1.8 of the textbook.)
5. Let S be an ordered set, $A \subset S$, and suppose that b is an upper bound for A. Prove that if $b \in A$ then $b = \sup(A)$.
6. Let S be an ordered set and A a nonempty subset that is bounded above. Suppose that $\sup(A)$ exists and that $\sup(A) \notin A$. Show that A is infinite, (Hint: it is enough to show that A contains a countably infinite subset.)
7. Let S be an ordered set and A a nonempty subset such that $\sup(A)$ exists. Suppose there is $B \subset A$ such that for each $x \in A$ there is $y \in B$ with $x \leq y$. Show that $\sup(B) = \sup(A)$.
8. Let $x \geq 0$ be a real number. Show that there exists $n \in \mathbb{N}$ such that $n - 1 \leq x < n$.
9. Prove the arithmetic-geometric mean inequality: for x, y positive and real, $\sqrt{xy} \leq \dfrac{x+y}{2}$ with equality if and only if $x = y$.
10. Let $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ be bounded and nonempty. Let $C = \{a + b \mid a \in A, b \in B\}$. Show that $\sup(C) = \sup(A) + \sup(B)$.