1. For each $n \in \mathbb{N}$ let $A_n = \{(n+1)k \ : \ k \in \mathbb{N}\}$.
   (a) Find $A_1 \cap A_2$
   (b) Find (with proof) $\bigcup_{n=1}^{\infty} A_n$
   (c) Find (with proof) $\bigcap_{n=1}^{\infty} A_n$
2. Prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ for every $n \in \mathbb{N}$.
3. Find (with proof) all $n \in \mathbb{N}$ such that $n^2 < 2^n$.
4. Let $f : A \to B$ and suppose that C, D are subsets of A. Prove that
   (a) $f(C \cup D) = f(C) \cup f(D)$
   (b) $f(C \cap D) \subset f(C) \cap f(D)$
   (c) Give an example in which $f(C \cap D) \subsetneq f(C) \cap f(D)$
5. Give an example of a countable collection of infinite sets $A_1, A_2, A_3, \ldots$ with the property that $A_i \cap A_j$ is infinite for every i and j but $\bigcap_{i=1}^{\infty} A_i$ is nonempty and finite.
6. Suppose $f : X \to Y$. Prove that $f^{-1}(f(A)) = A$ for every $A \subset X$ if and only if f is injective.
7. Give an example of a function f and sets A, X and Y such that $A \subset X$ yet $f^{-1}(f(A)) \neq A$.
8. Prove that if $|A \setminus B| = |B \setminus A|$ then $|A| = |B|$.
   Hint: Let $f : A \setminus B \to B \setminus A$ be a bijection. Define
   $$g(x) = \begin{cases} f(x) \text{ if } x \in A \setminus B \\ x \text{ if } x \in A \cap B \end{cases}$$
   and now show that $g : A \to B$ is a bijection.