Question 1: Let

$$f(x) = \begin{cases} 5 + x \text{ if } x < 1 \\ 1 - 4x \text{ if } x \geq 1 \end{cases}$$

(i) Find $f(2)$

$f(2) = 1 - 4(2) = \boxed{-7}$

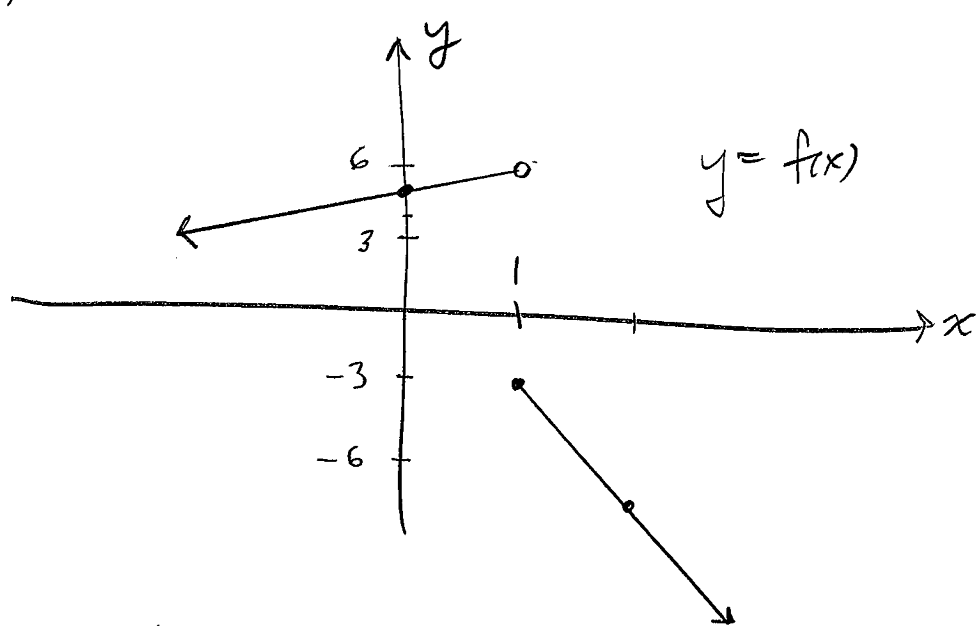
[1]

(ii) Find $f(0)$

$f(0) = 5 + 0 = \boxed{5}$

[1]

(iii) Graph $y = f(x)$



[3]

Question 2: Let $f(x) = 1 - x^2$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{[1-(x+h)^2] - [1-x^2]}{h}$$

$$= \frac{1 - x^2 - 2xh - h^2 - 1 + x^2}{h}$$

$$= \frac{h(-2x-h)}{h}$$

$$= \boxed{-2x-h}$$

[5]

Question 3: Let $f(x) = \sqrt{x-4}$ and $g(x) = \dfrac{2}{x}$.

(i) Find $(g \circ f)(x)$ = $g(f(x)) = \boxed{\dfrac{2}{\sqrt{x-4}}}$

[2]

(ii) Determine the domain of $(g \circ f)(x)$.

Require $x - 4 > 0$

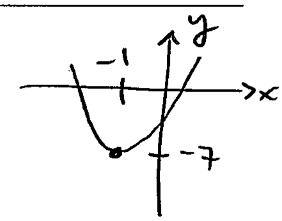
$\therefore\ x > 4$

$\therefore\ \boxed{(4, \infty)}$

[3]

Question 4: Let $f(x) = 2x^2 + 4x - 5$. Find

$$f(x) = 2\left[x^2 + 2x\right] - 5 = 2\left[(x+1)^2 - 1\right] - 5 = 2(x+1)^2 - 7$$



(i) the vertex (h, k) .

$\boxed{(-1, -7)}$

[2]

(ii) axis of symmetry.

$\boxed{x = -1}$

[1]

(iii) domain of f.

$\boxed{(-\infty, \infty)}$

[1]

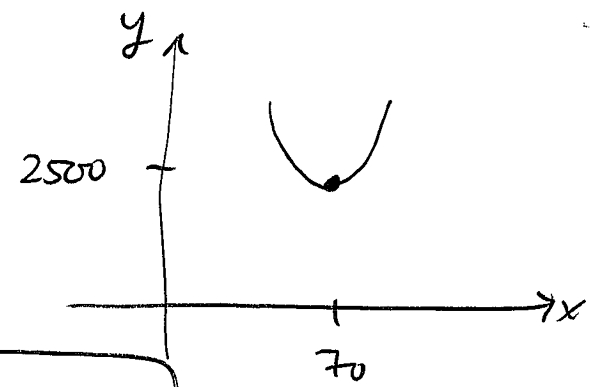
(iv) range of f.

$\boxed{[-7, \infty)}$

[1]

Question 5: The cost to produce x units of a certain product is given by $C(x) = x^2 - 140x + 7400$. How many units should be produced to minimize the cost?

$$C(x) = x^2 - 140x + 7400$$
$$= (x-70)^2 - 4900 + 7400$$
$$= (x-70)^2 + 2500$$



$\therefore$ $x = 70$ units should be produced

[5]

Question 6: Use synthetic division to determine the quotient: $\dfrac{5x^4 - 5x^3 + 2x^2 + x - 3}{x - 1}$

1	5	-5	2	1	-3
	↓	5	0	2	3
	5	0	2	3	10

$$\therefore \frac{5x^4 - 5x^3 + 2x^2 + x - 3}{x-1} = 5x^3 + 2x + 3$$

[5]

Question 7: Factor completely: $f(x) = 6x^3 - 13x^2 - 14x - 3$

Possible rational zeros: $\frac{p}{q}$: $p = \pm 1, \pm 3$

$q = \pm 1, \pm 2, \pm 3, \pm 6$

Try: $\frac{p}{q} = \frac{1}{1}$; no

$\frac{p}{q} = \frac{-1}{1}$; no

$\frac{p}{q} = \frac{3}{1}$; yes!

$$\begin{array}{c|cccc} 3 & 6 & -13 & -14 & -3 \\ & \downarrow & 18 & 15 & 3 \\ \hline & 6 & 5 & 1 & \boxed{0} \end{array}$$

$\therefore f(x) = (x-3)(6x^2+5x+1)$

$= (x-3)(6x^2+3x+2x+1)$

$= (x-3)(3x(2x+1)+(2x+1))$

$= \boxed{(x-3)(2x+1)(3x+1)}$

[5]

Question 8: Determine the value of k so that $\frac{x^3 - 3x^2 + kx - 4}{x - 2}$ has remainder 3.

$$\begin{array}{c|cccc} 2 & 1 & -3 & k & -4 \\ & \downarrow & 2 & -2 & 2k-4 \\ \hline & 1 & -1 & k-2 & \boxed{3} \end{array}$$

$\therefore -4 + (2k-4) = 3$

$2k = 3 + 8$

$\boxed{k = \frac{11}{2}}$

[5]

Question 9: For this question use the polynomial function

$$f(x) = x^2(x-3)^2(x+1)$$

(i) State the zeros of f.

$x = 0, 3, -1$

[2]

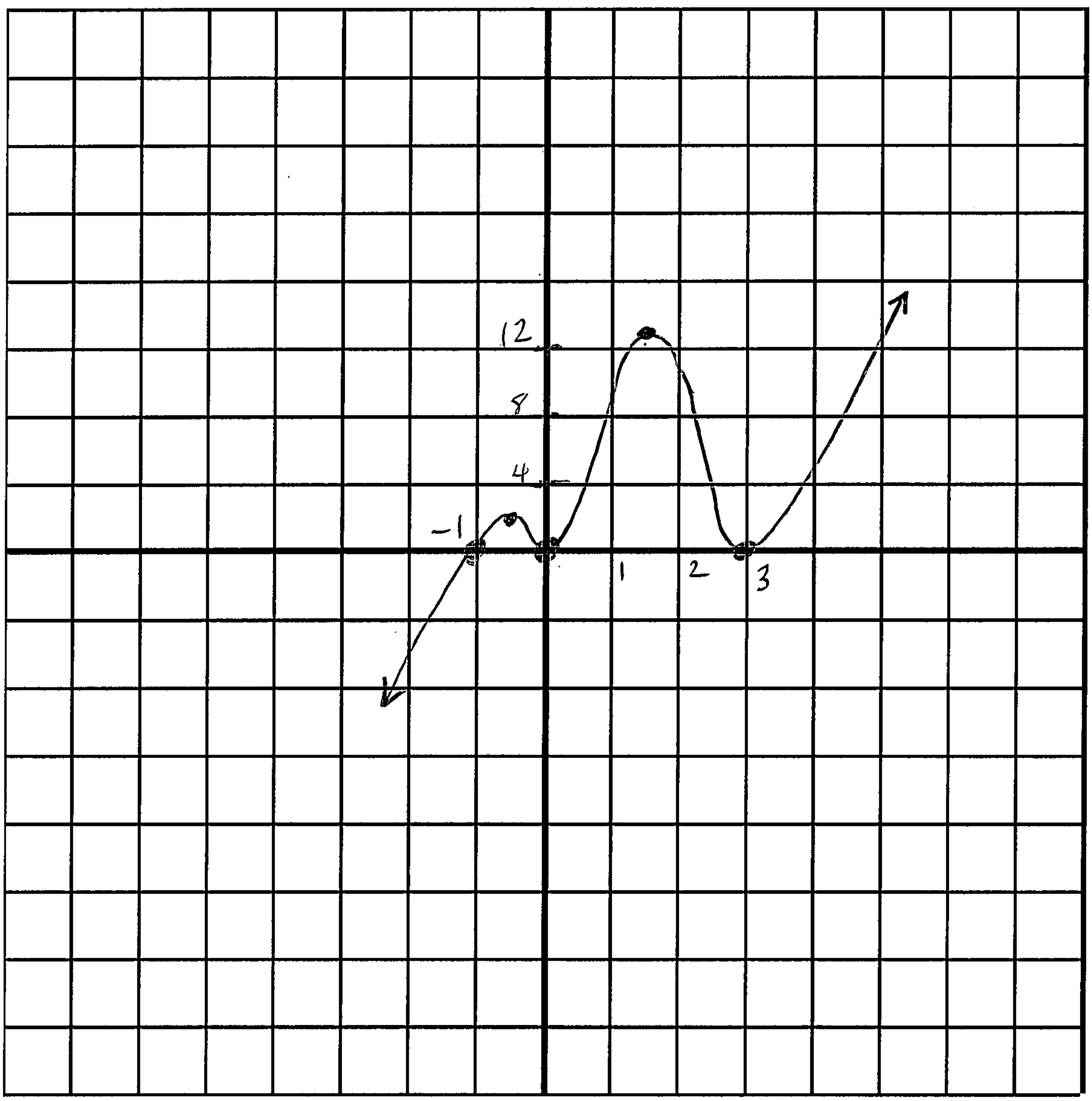
(ii) Determine the y-intercept.

$y = f(0) = 0$

[2]

(iii) Sketch an approximate graph of $y = f(x)$ showing the correct behaviour of the curve at the zeros, the y-intercept, and the correct end behaviour.

$f(-\frac{1}{2}) \doteq 1.5$; $f(\frac{3}{2}) \doteq 12.7$



[6]