Question 1: Let

$$f(x) = \begin{cases} 5 - x \text{ if } x < 1 \\ 1 + 4x \text{ if } x \geq 1 \end{cases}$$

(i) Find $f(2)$

$f(2) = 1+4(2) = \boxed{9}$

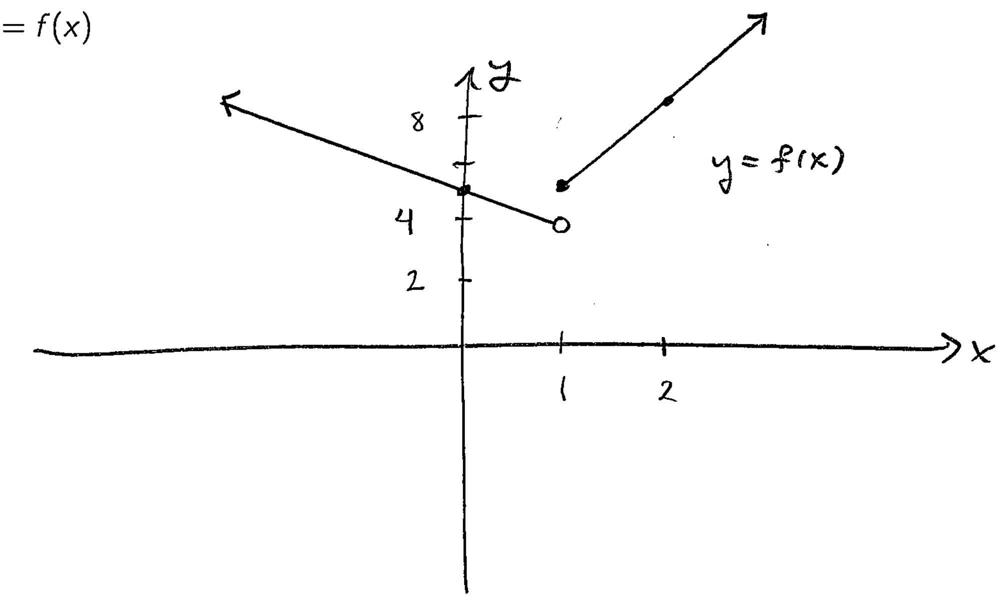
[1]

(ii) Find $f(0)$

$f(0) = 5-0 = \boxed{5}$

[1]

(iii) Graph $y = f(x)$



[3]

Question 2: Let $f(x) = 1 + x^2$. Find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{[1+(x+h)^2] - [1+x^2]}{h}$$

$$= \frac{\cancel{1}+\cancel{x^2}+2xh+h^2\cancel{-1}-\cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \boxed{2x+h}$$

[5]

Question 3: Let $f(x) = \sqrt{x+4}$ and $g(x) = \dfrac{-2}{x}$.

(i) Find $(g \circ f)(x) = g(f(x)) = \boxed{\dfrac{-2}{\sqrt{x+4}}}$

[2]

(ii) Determine the domain of $(g \circ f)(x)$.

Require $x+4 > 0$

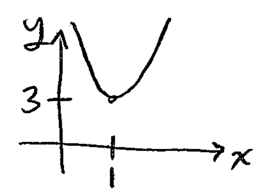
$\therefore \quad x > -4$

$\therefore \quad \boxed{(-4, \infty)}$

[3]

Question 4: Let $f(x) = 2x^2 - 4x + 5$. Find

$$\begin{aligned} f(x) &= 2\left[x^2 - 2x\right] + 5 \\ &= 2\left[(x-1)^2 - 1\right] + 5 \\ &= 2(x-1)^2 + 3 \end{aligned}$$



(i) the vertex (h, k) .

$\boxed{(1, 3)}$

[2]

(ii) axis of symmetry.

$\boxed{x = 1}$

[1]

(iii) domain of f.

$\boxed{(-\infty, \infty)}$

[1]

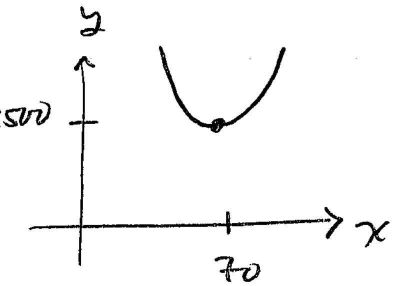
(iv) range of f.

$\boxed{[3, \infty)}$

[1]

Question 5: The cost to produce x units of a certain product is given by $C(x) = x^2 - 140x + 7400$. How many units should be produced to minimize the cost?

$$\begin{aligned} C(x) &= x^2 - 140x + 7400 \\ &= (x-70)^2 - 4900 + 7400 \\ &= (x-70)^2 + 2500. \end{aligned}$$



$\boxed{\therefore\ x = 70 \text{ units should be produced}}$

[5]

Question 6: Use synthetic division to determine the quotient: $\dfrac{5x^4 + 5x^3 + 2x^2 - x - 3}{x+1}$

$$\begin{array}{r|rrrrr} -1 & 5 & 5 & 2 & -1 & -3 \\ & \downarrow & -5 & 0 & -2 & 3 \\ \hline & 5 & 0 & 2 & -3 & \boxed{0} \end{array}$$

$$\therefore \frac{5x^4 + 5x^3 + 2x^2 - x - 3}{x+1} = \boxed{5x^3 + 2x - 3}$$

[5]

Question 7: Factor completely: $f(x) = 6x^3 + 13x^2 - 14x + 3$

Possible rational zeros $\frac{p}{q}$: $p = \pm 1, \pm 3$
$q = \pm 1, \pm 2, \pm 3, \pm 6.$

Try: $\frac{p}{q} = \frac{1}{1}$: no

$\frac{p}{q} = \frac{-1}{1}$: no

$\frac{p}{q} = \frac{3}{1}$: no

$\frac{p}{q} = \frac{-3}{1}$: Yes!

$$\begin{array}{r|rrrr} -3 & 6 & 13 & -14 & 3 \\ & \downarrow & -18 & 15 & -3 \\ \hline & 6 & -5 & 1 & \boxed{0} \end{array}$$

$$\therefore f(x) = (x+3)(6x^2 - 5x + 1)$$
$$= (x+3)(6x^2 - 3x - 2x + 1)$$
$$= (x+3)[3x(2x-1) - (2x-1)]$$
$$= \boxed{(x+3)(2x-1)(3x-1)}$$

[5]

Question 8: Determine the value of k so that $\dfrac{x^3 - 3x^2 + kx - 4}{x - 2}$ has remainder 5.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & k & -4 \\ & \downarrow & 2 & -2 & (2k-4) \\ \hline & 1 & -1 & k-2 & \boxed{5} \end{array}$$

$$\therefore -4 + (2k-4) = 5$$
$$2k = 5 + 8$$
$$\boxed{k = \frac{13}{2}}$$

[5]

Question 9: For this question use the polynomial function

$$f(x) = x^2(x-3)^3(x+1)$$

(i) State the zeros of f.

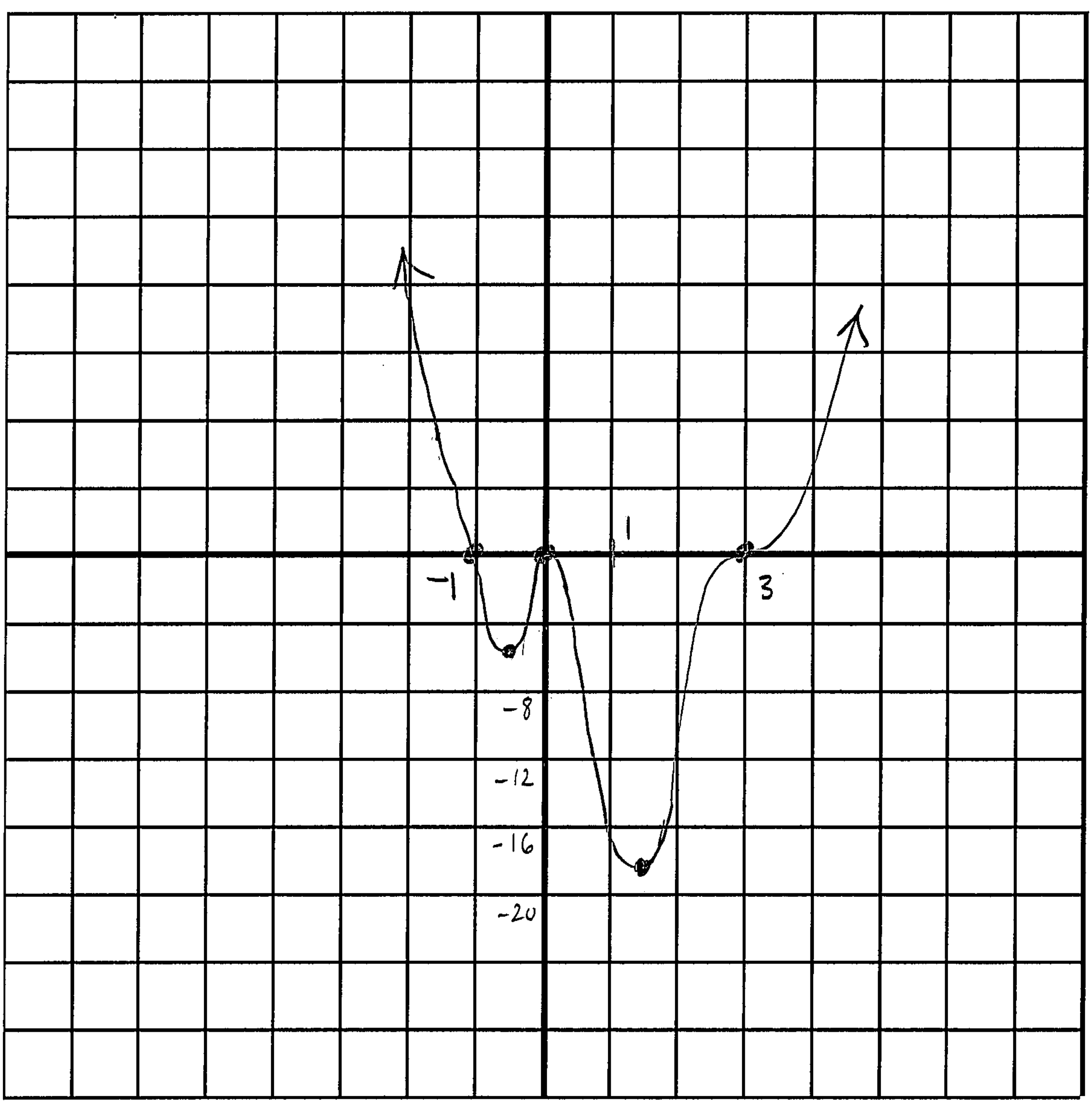
$x = 0$, $x = 3$, $x = -1$ **[2]**

(ii) Determine the y-intercept.

$y = f(0) = 0$ **[2]**

(iii) Sketch an approximate graph of $y = f(x)$ showing the correct behaviour of the curve at the zeros, the y-intercept, and the correct end behaviour.

$f(-\frac{1}{2}) \doteq -5.4$; $f(\frac{3}{2}) \doteq -19$



[6]