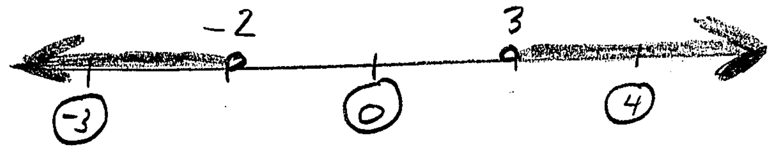
Question 1: Solve the following inequalities. State your answers using interval notation.

(a) $x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$

$x - 3 = 0$; $x + 2 = 0$

$x = 3$ $\quad x = -2$



$(x-3)(x+2)$: $+$ $\quad -$ $\quad +$

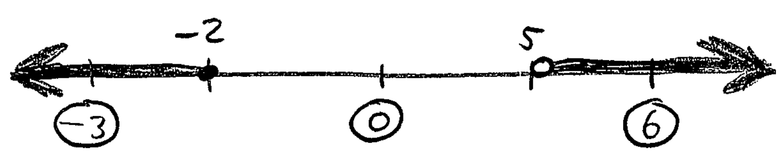
$\therefore$ $x^2 - x - 6 > 0$ on $\boxed{(-\infty, -2) \cup (3, \infty)}$,

[5]

(b) $\dfrac{3x+6}{x-5} \geq 0$

$3x + 6 = 0$; $x - 5 = 0$

$x = -2$ $\quad x = 5$



$\dfrac{3x+6}{x-5}$: $+$ $\quad -$ $\quad +$

$\therefore$ $\dfrac{3x+6}{x-5} \geq 0$ on $\boxed{(-\infty, -2] \cup (5, \infty)}$.

[5]

Question 2:

(a) Find the distance between the points $P(-4, 3)$ and $Q(-2, 5)$.

$$d(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(5-3)^2 + (-2-(-4))^2}$$

$$= 2^2$$

$$= \boxed{4}$$

[3]

(b) Find the midpoint of the line segment joining the points $P(-4, 3)$ and $Q(-2, 5)$.

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left(\frac{-4+(-2)}{2}, \frac{3+5}{2} \right)$$

$$= \boxed{(-3,4)}$$

[3]

(c) Suppose $Q(-2, 5)$ is the midpoint of the line segment joining $P(-4, 3)$ and some other point $R(a, b)$. Find the coordinates a, b of R.

$$(-2,5) = \left(\frac{-4+a}{2}, \frac{3+b}{2} \right)$$

$$\therefore -2 = \frac{-4+a}{2} \qquad 5 = \frac{3+b}{2}$$

$$-4 = -4+a \qquad 10 = 3+b$$

$$a = 0 \qquad b = 7$$

$$\boxed{\therefore (a,b) = (0,7)}$$

[4]

Question 3:

(a) Find the equation of the circle with center $(2, -3)$ and radius $3/4$.

$$\boxed{(x-2)^2 + (y+3)^2 = \frac{9}{16}}$$

[2]

(b) Determine (i) the center and (ii) the radius of the circle having equation

$$x^2 + y^2 + 8x - 6y + 16 = 0 \ .$$

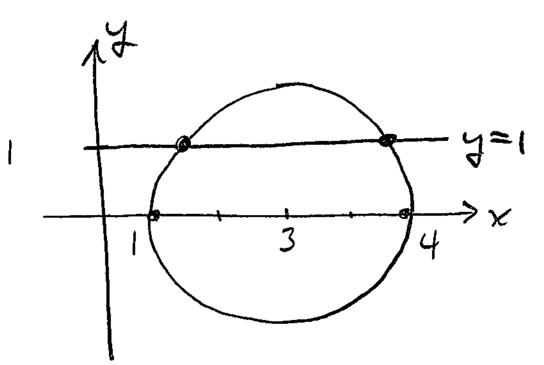
$$x^2 + 8x + y^2 - 6y + 16 = 0$$
$$(x+4)^2 - 16 + (y-3)^2 - 9 + 16 = 0$$
$$(x+4)^2 + (y-3)^2 = 9$$

$\therefore$ (i) Center is $(-4, 3)$

(ii) Radius is $r = 3$

[4]

(c) Find all points of intersection of the line $y = 1$ with the circle of radius 2 and center $(3, 0)$.



Let $(a, 1)$ be a point of intersection.

Circle has equation

$$(y-0)^2 + (x-3)^2 = 2^2$$
$$\therefore (1-0)^2 + (a-3)^2 = 2^2$$
$$1 + (a-3)^2 = 4$$
$$(a-3)^2 = 3$$
$$a - 3 = \pm\sqrt{3}$$
$$a = 3 \pm \sqrt{3}.$$

$\therefore$ points are $(3+\sqrt{3}, 1)$, $(3-\sqrt{3}, 1)$

[4]

Question 4:

(a) Determine the domain of $f(x) = \frac{\sqrt{4x+1}}{x}$. State your anwer using interval notation.

Because of $\sqrt{\ }$: $4x+1 \geq 0$

$x \geq \frac{-1}{4}$

Because of denominator: $x \neq 0$.

$\therefore$ domain is $\boxed{[-\frac{1}{4}, 0) \cup (0, \infty)}$

[3]

(b) Let $g(x) = x^2 - 3x + 2$. Find and simplify $g(2a+1)$.

$g(2a+1) = (2a+1)^2 - 3(2a+1) + 2$

$= 4a^2 + 4a + 1 - 6a - 3 + 2$

$= 4a^2 - 2a$

$= \boxed{2a(2a-1)}$

[3]

(c) Determine the value of a if the point $(3, 2)$ is on the graph of $f(x) = \frac{1}{2x - a}$.

$f(3) = 2$ so $\frac{1}{2(3)-a} = 2$

$\frac{1}{2} = 6 - a$

$a = 6 - \frac{1}{2}$

$\boxed{a = \frac{11}{2}}$

[4]

Question 5:

(a) Find the slope and y-intercept of the line $5x - 2y = 10$.

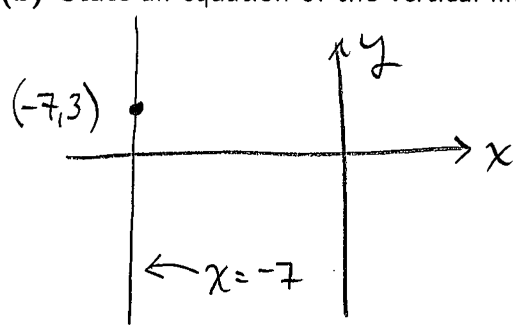
$$-2y = -5x + 10$$

$$y = \frac{5}{2}x - 5$$

$$\boxed{\therefore\ m = \frac{5}{2} \quad y\text{-intercept is } (0,-5)}$$

[2]

(b) State an equation of the vertical line through the point $(-7, 3)$.



$$\boxed{x = -7}$$

[2]

(c) Determine an equation of the line through the points $(-1, 3)$ and $(3, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{3-(-1)} = \frac{1}{4}$$

$$\therefore\ y - y_1 = m(x - x_1)$$

$$\boxed{y - 3 = \frac{1}{4}(x+1)}$$

or $\boxed{y = \frac{1}{4}x + \frac{13}{4}}$

or $\boxed{x - 4y = -13}$

[3]

(d) Determine an equation of the line through the point $P(1, 6)$ that is perpendicular to the line $3x + 5y = 1$.

$$3x + 5y = 1$$

$$5y = -3x + 1$$

$$y = -\frac{3}{5}x + \frac{1}{5}$$

$\therefore$ slope of line we want is $m = \frac{(-1)}{\left(-\frac{3}{5}\right)} = \frac{5}{3}$

$$\boxed{\therefore \text{ equation is } y - 6 = \frac{5}{3}(x-1)}$$

or $\boxed{y = \frac{5}{3}x + \frac{13}{3}}$

or $\boxed{5x - 3y = -13}$

[3]