

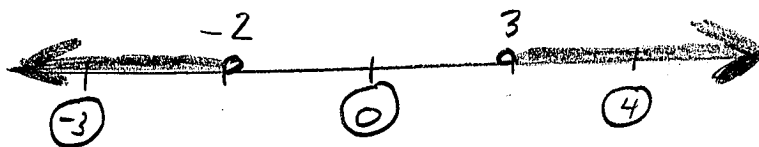
Question 1: Solve the following inequalities. State your answers using interval notation.

(a) $x^2 - x - 6 > 0$

$$(x-3)(x+2) > 0$$

$$x-3=0 ; x+2=0$$

$$x=3 \quad x=-2$$



$$(x-3)(x+2) : \quad \oplus \quad \ominus \quad \oplus$$

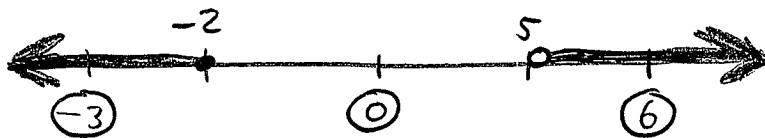
$$\therefore x^2 - x - 6 > 0 \quad \text{on} \quad \boxed{(-\infty, -2) \cup (3, \infty)}$$

[5]

(b) $\frac{3x+6}{x-5} \geq 0$

$$3x+6=0 ; x-5=0$$

$$x=-2 \quad x=5$$



$$\frac{3x+6}{x-5} : \quad \oplus \quad \ominus \quad \oplus$$

$$\therefore \frac{3x+6}{x-5} \geq 0 \quad \text{on} \quad \boxed{(-\infty, -2] \cup (5, \infty)}$$

[5]

Question 2:

- (a) Find the distance between the points
- $P(-4, 3)$
- and
- $Q(-2, 5)$
- .

$$\begin{aligned}
 d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(5 - 3)^2 + (-2 - (-4))^2} \\
 &= \sqrt{2^2 + 2^2} \\
 &= \cancel{4} 2\sqrt{2}
 \end{aligned}$$

[3]

- (b) Find the midpoint of the line segment joining the points
- $P(-4, 3)$
- and
- $Q(-2, 5)$
- .

$$\begin{aligned}
 M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-4 + (-2)}{2}, \frac{3 + 5}{2} \right) \\
 &= \boxed{(-3, 4)}
 \end{aligned}$$

[3]

- (c) Suppose
- $Q(-2, 5)$
- is the midpoint of the line segment joining
- $P(-4, 3)$
- and some other point
- $R(a, b)$
- . Find the coordinates
- a, b
- of
- R
- .

$$(-2, 5) = \left(\frac{-4 + a}{2}, \frac{3 + b}{2} \right)$$

$$\begin{aligned}
 \therefore -2 &= \frac{-4 + a}{2} & \left\{ \begin{array}{l} 5 = \frac{3 + b}{2} \\ 10 = 3 + b \\ b = 7 \end{array} \right. \\
 -4 &= -4 + a \\
 a &= 0
 \end{aligned}$$

$$\therefore (a, b) = (0, 7)$$

[4]

Question 3:

- (a) Find the equation of the circle with center
- $(2, -3)$
- and radius
- $3/4$
- .

$$(x-2)^2 + (y+3)^2 = \frac{9}{16}$$

[2]

- (b) Determine (i) the center and (ii) the radius of the circle having equation

$$x^2 + y^2 + 8x - 6y + 16 = 0.$$

$$x^2 + 8x + y^2 - 6y + 16 = 0$$

$$(x+4)^2 - 16 + (y-3)^2 - 9 + 16 = 0$$

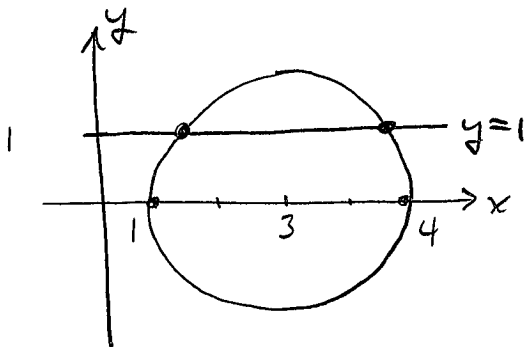
$$(x+4)^2 + (y-3)^2 = 9$$

$$\therefore \text{(i) Center is } (-4, 3)$$

$$\text{(ii) Radius is } r = 3$$

[4]

- (c) Find all points of intersection of the line
- $y = 1$
- with the circle of radius 2 and center
- $(3, 0)$
- .

Let $(a, 1)$ be a point of intersection.

Circle has equation

$$(y-0)^2 + (x-3)^2 = 2^2$$

$$\therefore (1-0)^2 + (a-3)^2 = 2^2$$

$$1 + (a-3)^2 = 4$$

$$(a-3)^2 = 3$$

$$a-3 = \pm \sqrt{3}$$

$$a = 3 \pm \sqrt{3}$$

$$\therefore \text{points are } (3+\sqrt{3}, 1), (3-\sqrt{3}, 1)$$

[4]

Question 4:

- (a) Determine the domain of $f(x) = \frac{\sqrt{4x+1}}{x}$. State your answer using interval notation.

$$\text{Because of } \sqrt{\quad} : 4x+1 \geq 0$$

$$x \geq -\frac{1}{4}$$

$$\text{Because of denominator: } x \neq 0.$$

$$\therefore \text{ domain is } \left[-\frac{1}{4}, 0\right) \cup (0, \infty)$$

[3]

- (b) Let $g(x) = x^2 - 3x + 2$. Find and simplify $g(2a+1)$.

$$g(2a+1) = (2a+1)^2 - 3(2a+1) + 2$$

$$= 4a^2 + 4a + 1 - 6a - 3 + 2$$

$$= 4a^2 - 2a$$

$$= \boxed{2a(2a-1)}$$

[3]

- (c) Determine the value of a if the point $(3, 2)$ is on the graph of $f(x) = \frac{1}{2x-a}$.

$$f(3) = 2 \quad \text{so} \quad \frac{1}{2(3)-a} = 2$$

$$\frac{1}{2} = 6-a$$

$$a = 6 - \frac{1}{2}$$

$$\boxed{a = \frac{11}{2}}$$

[4]

Question 5:

- (a) Find the slope and y-intercept of the line
- $5x - 2y = 10$
- .

$$-2y = -5x + 10$$

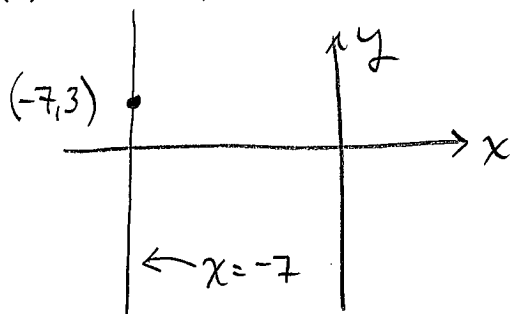
$$y = \frac{5}{2}x - 5$$

$$\therefore m = \frac{5}{2}$$

y-intercept is $(0, -5)$

[2]

- (b) State an equation of the vertical line through the point
- $(-7, 3)$
- .



$$x = -7$$

[2]

- (c) Determine an equation of the line through the points
- $(-1, 3)$
- and
- $(3, 4)$
- .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 3}{3 - (-1)}$$

$$= \frac{1}{4}$$

$$\therefore y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{4}(x + 1)$$

$$\text{or } y = \frac{1}{4}x + \frac{13}{4}$$

$$\text{or } x - 4y = -13$$

[3]

- (d) Determine an equation of the line through the point
- $P(1, 6)$
- that is perpendicular to the line
- $3x + 5y = 1$
- .

$$3x + 5y = 1$$

$$5y = -3x + 1$$

$$y = -\frac{3}{5}x + \frac{1}{5}$$

\therefore slope of line we want

$$\text{is } m = \frac{(-1)}{\left(-\frac{3}{5}\right)} = \frac{5}{3}$$

$$\therefore \text{equation is } y - 6 = \frac{5}{3}(x - 1)$$

$$\text{or } y = \frac{5}{3}x + \frac{13}{3}$$

$$\text{or } 5x - 3y = -13$$

[3]