Question 1:

(a) Simplify: $\dfrac{4(x^2-1)^3 + 8x(x^2-1)^4}{16(x^2-1)^3}$

$= \dfrac{4(x^2-1)^3[1 + 2x(x^2-1)]}{4 \cdot 16(x^2-1)^3}$

$= \boxed{\dfrac{2x^3 - 2x + 1}{4}}$

[3]

(b) Simplify to a single real number: $\dfrac{r^{-1}+q^{-1}}{r^{-1}-q^{-1}} \cdot \dfrac{r-q}{r+q}$

$= \dfrac{r + rq^{-1} - r^{-1}q - q}{r - rq^{-1} + r^{-1}q - q}$

$= \dfrac{(rq^{-1} - r^{-1}q)}{-(rq^{-1} - r^{-1}q)}$

$= \boxed{-1}$

[4]

(c) Rationalize the denominator.

$\dfrac{a}{\sqrt{a+b}+1} \cdot \dfrac{\sqrt{a+b}-1}{\sqrt{a+b}-1}$

$= \boxed{\dfrac{a\left(\sqrt{a+b}-1\right)}{a+b-1}}$

[3]

Question 2:

(a) Solve for x:

$$4(2x + 1) = 6 - (2x - 4)$$

$$8x + 4 = 6 - 2x + 4$$

$$10x = 6.$$

$$x = \frac{6}{10}$$

$$\boxed{x = \frac{3}{5}}$$

[3]

(b) Solve for x:

$$\frac{1}{15}(2x + 5) = \frac{x - 2}{9}$$

$$9(2x + 5) = 15(x - 2)$$

$$18x + 45 = 15x - 30$$

$$3x = -75$$

$$\boxed{x = -25}$$

[3]

(c) $750 is invested for 36 months at a certain rate of simple interest. At the end of the 36 months the total value of the investment is $840. What is the rate of simple interest? (Express your answer as a percentage rounded to two decimal places.)

$P = \$750$ $A = P(1 + rt)$

$t = \frac{36}{12} = 3 \text{ yrs.}$ $r = \frac{\left(\frac{A}{P} - 1\right)}{t}$

$A = \$840$

$r = ?$ $= \frac{\left(\frac{840}{750} - 1\right)}{3}$

$= 0.04$

$= \boxed{4.00\%}$

[4]

Question 3:

(a) $10,000 is split between two investments: one pays 3% simple interest and the second pays 5% simple interest. After two years the investments have earned $840 interest in total. How much was originally invested at the 3% rate? (Round your answer to the nearest dollar.)

Let x = amount invested at 3%

∴ $10000 - x$ = amount invested at 5%.

$t = 2$ for both.

Using $I = Prt$:

$(x)(0.03)(2) + (10000 - x)(0.05)(2) = 840$

$(0.06)x + 1000 - (0.1)x = 840$

$(-0.04)x = -160$

∴ $x = 4000$

∴ $\$4000$ was invested at 3%

[5]

(b) Mary and Janet run a race. Mary runs at 14 km/h while Janet runs at 10 km/h. If they start at the same time, how long will it take them to be 1 km apart? (Round your answer to one decimal place.)

Let t be the required time in hours.

Using $d = rt$, in t hours:

Mary runs $14t$ km,

Janet runs $10t$ km.

So $14t - 10t = 1$

$4t = 1$

$t = \frac{1}{4}$ hr.

∴ After $\frac{1}{4}$ hr they will be 1 km apart

[5]

Question 4:

(a) Solve for x:

$$-3x^2 + 4x = -2$$

$$3x^2 - 4x - 2 = 0$$

$$a = 3, \quad b = -4, \quad c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{40}}{6}$$

$$= \frac{4 \pm 2\sqrt{10}}{6}$$

$$= \frac{2 \pm \sqrt{10}}{3}$$

$$\therefore x = \frac{2 + \sqrt{10}}{3}, \quad \frac{2 - \sqrt{10}}{3}$$

[5]

(b) A picture is of size 10 inches by 12 inches and we wish to put a border around the outside. The border will have an area of 48 square inches and have the same width on all four sides. Determine the width of the border.

Let $x =$ width of border.

We have $(10 + 2x)(12 + 2x) - (10)(12) = 48$

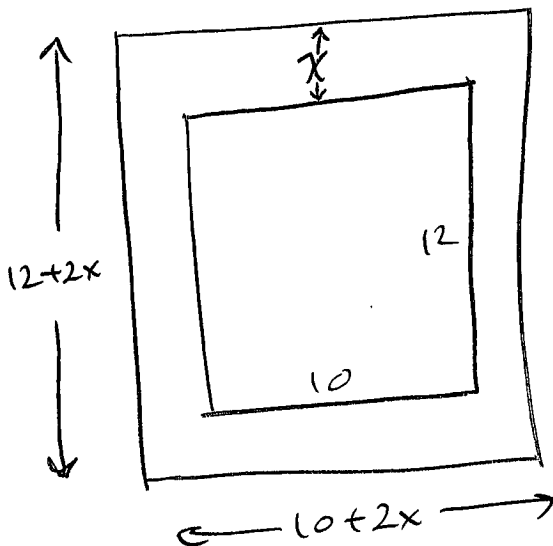$$120 + 24x + 20x + 4x^2 - 120 = 48$$

$$4x^2 + 44x - 48 = 0$$

$$4(x^2 + 11x - 12) = 0$$

$$4(x - 1)(x + 12) = 0$$

$$\therefore x = 1, \quad \cancel{x = -12} \quad \text{since } x > 0.$$

$\therefore$ border should be 1 inch wide

[5]

Question 5:

(a) Solve for x:

$$\left(\sqrt{x+2}\right)^2 = \left(1 - \sqrt{3x+7}\right)^2$$

$$x + 2 = 1 - 2\sqrt{3x+7} + 3x+7$$

$$2x + 6 = 2\sqrt{3x+7}$$

$$\frac{\cancel{2}(x+3)}{\cancel{2}} = \sqrt{3x+7}$$

$$(x+3)^2 = 3x+7$$

$$x^2 + 6x+9 = 3x+7$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, \quad \cancel{x = -1}.$$

$\therefore$ $x = -2$ is only solution

Check! $\underline{x=-2}$: $\quad \sqrt{-2+2} \stackrel{?}{=} 1 - \sqrt{3(-2)+7}$ $\quad \Big\{$ $\underline{x=-1}$: $\sqrt{-1+2} \stackrel{?}{=} 1 - \sqrt{3(-1)+7}$

$$0 = 0 \checkmark \qquad\qquad\qquad 1 \stackrel{?}{=} -1 \quad X$$

[5]

(b) Solve for x:

$$\frac{2x+1}{x-2} + \frac{3}{x} = \frac{-6}{x^2-2x} \qquad\Big\} \quad x \neq 2, \quad x \neq 0.$$

$$x(x-2)\left[\frac{2x+1}{x-2} + \frac{3}{x}\right] = \left[\frac{-6}{x(x-2)}\right] x(x-2)$$

$$x(2x+1) + (x-2)(3) = -6$$

$$2x^2 + x + 3x - \cancel{6} = -\cancel{6}$$

as noted above $\qquad 2x(x+2) = 0$

$$\boxed{\cancel{x=0}}, \quad x = -2$$

$\therefore$ $x = -2$ is only solution

Check! $\underline{x=-2}$: $\quad \dfrac{2(-2)+1}{-2-2} + \dfrac{3}{-2} \stackrel{?}{=} \dfrac{-6}{(-2)^2 - 2(-2)}$

$$\frac{3}{4} - \frac{3}{2} \stackrel{?}{=} -\frac{3}{4}$$

$$-\frac{3}{4} = \frac{-3}{4} \checkmark$$

[5]