Question 1:

(a) Simplify to a single real number: $\dfrac{r^{-1}+q^{-1}}{r^{-1}-q^{-1}}\cdot\dfrac{r-q}{r+q}$ $=\dfrac{\cancel{r}+rq^{-1}-r^{-1}q-\cancel{r}}{\cancel{r}-rq^{-1}+r^{-1}q-\cancel{r}}$

$$=\dfrac{\cancel{(rq^{-1}-r^{-1}q)}}{-\cancel{(rq^{-1}-r^{-1}q)}}$$

$$=\boxed{-1}$$

[3]

(b) Simplify: $\dfrac{4(x^2-1)^3+8x(x^2-1)^4}{16(x^2-1)^3}$ $=\dfrac{\cancel{4}\cancel{(x^2-1)^3}\left[1+2x(x^2-1)\right]}{4\cdot\cancel{16}\cancel{(x^2-1)^3}}$

$$=\boxed{\dfrac{2x^3-2x+1}{4}}$$

[4]

(c) Rationalize the denominator.

$$\dfrac{a}{\sqrt{a+b}-1}\cdot\dfrac{\sqrt{a+b}+1}{\sqrt{a+b}+1}$$

$$=\boxed{\dfrac{a\,(\sqrt{a+b}+1)}{a+b-1}}$$

[3]

Question 2:

(a) Solve for x:

$$4(-2x + 1) = 6 - (2x - 4)$$

$$-8x + 4 = 6 - 2x + 4$$

$$-6x = 6$$

$$x = -\frac{6}{6}$$

$$\boxed{x = -1}$$

[3]

(b) Solve for x:

$$\frac{1}{15}(2x + 5) = \frac{x + 2}{9}$$

$$9(2x + 5) = 15(x + 2)$$

$$18x + 45 = 15x + 30$$

$$3x = -15$$

$$\boxed{x = -5}$$

[3]

(c) \$750 is invested for 36 months at a certain rate of simple interest. At the end of the 36 months the total value of the investment is \$840. What is the rate of simple interest? (Express your answer as a percentage rounded to two decimal places.)

$$P = \$750 \qquad\qquad A = P(1 + rt)$$

$$t = \frac{36}{12} = 3 \text{ yrs.} \qquad \therefore r = \frac{\left(\frac{A}{P} - 1\right)}{t}$$

$$A = \$840$$

$$r = ? \qquad\qquad = \frac{\left(\frac{840}{750} - 1\right)}{3}$$

$$= 0.04 = \boxed{4.00\%}$$

[4]

Question 3:

(a) $10,000 is split between two investments: one pays 3% simple interest and the second pays 5% simple interest. After two years the investments have earned $840 interest in total. How much was originally invested at the 5% rate? (Round your answer to the nearest dollar.)

Let X = amount invested at 5%.

$\therefore$ $10000 - X$ = amount invested at 3%.

$t = 2$ for both investments.

Using $I = Prt$:

$$(X)(0.05)(2) + (10000 - X)(0.03)(2) = 840.$$

$$\therefore (0.1)X + 600 - (0.06)X = 840$$

$$(0.04)X = 240$$

$$\boxed{X = 6000}.$$

$\therefore$ \$6000 was invested at 5%.   [5]

(b) Mary and Janet run a race. Mary runs at 14 km/h while Janet runs at 10 km/h. If they start at the same time, how long will it take them to be 3 km apart? (Round your answer to one decimal place.)

Let t be the required time in hours.

Using $d = rt$, in t hours Mary runs $14t$ km, while Janet runs $10t$ km.

So $14t - 10t = 3$

$$4t = 3$$

$$\boxed{t = \frac{3}{4} \text{ hr}}.$$

$\therefore$ After $\frac{3}{4}$ hr they will be 3 km apart.

[5]

Question 4:

(a) Solve for x:

$$-3x^2 + 4x = -1$$

$$3x^2 - 4x - 1 = 0$$

$$a = 3, \quad b = -4, \quad c = -1.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{6} \qquad \Rightarrow = \frac{2 \pm \sqrt{7}}{3}$$

$$= \frac{4 \pm \sqrt{28}}{6} \qquad \boxed{\therefore x = \frac{2 + \sqrt{7}}{3}, \quad \frac{2 - \sqrt{7}}{3}}$$

$$= \frac{4 \pm 2\sqrt{7}}{6}$$

[5]

(b) A picture is of size 10 inches by 12 inches and we wish to put a border around the outside. The border will have an area of 48 square inches and have the same width on all four sides. Determine the width of the border.

Let x = width of border.

We have $(10 + 2x)(12 + 2x) - (10)(12) = 48$

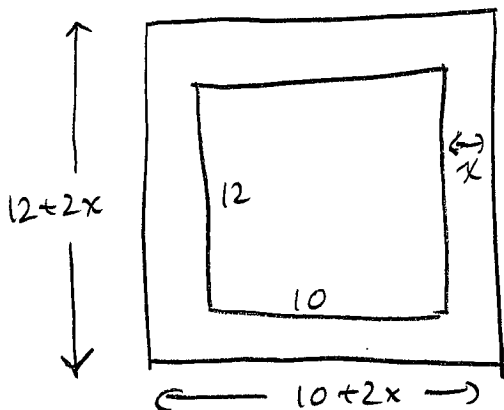$$120 + 24x + 20x + 4x^2 - 120 = 48$$

$$4x^2 + 44x - 48 = 0$$

$$4(x^2 + 11x - 12) = 0$$

$$4(x - 1)(x + 12) = 0$$

$$\therefore x = 1, \quad \cancel{x = -12}$$

$\uparrow$ since $x \geq 0$.

$$\boxed{\therefore \text{border should be 1 inch wide}}$$

[5]

Question 5:

(a) Solve for x:

$$\frac{2x+1}{x-2} + \frac{3}{x} = \frac{-6}{x^2 - 2x} \qquad \Big\{ \; x \neq 0, \; x \neq 2.$$

$$x(x-2)\left[\frac{2x+1}{x-2} + \frac{3}{x}\right] = \left[\frac{-6}{x(x-2)}\right] x(x-2)$$

$$x(2x+1) + 3(x-2) = -6$$

$$2x^2 + x + 3x - 6 + 6 = 0$$

as noted above $\longrightarrow$ $$2x(x+2) = 0$$

$$\boxed{\cancel{x=0}} \; , \; \boxed{x = -2}$$

Check! $\frac{2x+1}{x-2} + \frac{3}{x} = \frac{-6}{x^2-2x}$

$\frac{2(-2)+1}{-2-2} + \frac{3}{-2} \overset{?}{=} \frac{-6}{(-2)^2 - 2(-2)}$ $\Big\}\Rightarrow$

$\frac{-3}{-4} - \frac{3}{2} \overset{?}{=} \frac{-6}{8}$

$\frac{3-6}{4} \overset{?}{=} -\frac{3}{4}$

$\frac{-3}{4} = \frac{-3}{4}$ ✓

[5]

(b) Solve for x:

$$\left(\sqrt{x+2}\right)^2 = \left(1 - \sqrt{3x+7}\right)^2$$

$$x+2 = 1 - 2\sqrt{3x+7} + 3x+7$$

$$\text{so} \; \frac{2x+6}{2} = \frac{2\sqrt{3x+7}}{2}$$

$$\left(x+3\right)^2 = \left(\sqrt{3x+7}\right)^2$$

$$x^2 + 6x + 9 = 3x+7$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2 \; , \; \boxed{\cancel{x = -1}} \; \Longleftarrow$$

∴ x = -2 is
the only solution!

Check!

$x = -2$: $\sqrt{-2+2} \overset{?}{=} 1 - \sqrt{3(-2)+7}$

$0 = 0$ ✓

$\Big\{ \; x = -1$: $\sqrt{-1+2} \overset{?}{=} 1 - \sqrt{3(-1)+7}$

$1 \overset{?}{=} -1$ ✗

[5]