

Question 1: For this question use the following sets:

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}, \quad M = \{0, 2, 4, 6, 8\}$$

$$N = \{1, 3, 5, 7, 9, 11, 13\}, \quad Q = \{0, 2, 4, 6, 8, 10, 12\}, \quad R = \{0, 1, 2, 3, 4\}$$

Using these, find each of the following:

(a) $M \cup Q = \{0, 2, 4, 6, 8\} \cup \{0, 2, 4, 6, 8, 10, 12\}$

$$= \boxed{\{0, 2, 4, 6, 8, 10, 12\}}$$

[2]

(b) $Q \cap R' = \{0, 2, 4, 6, 8, 10, 12\} \cap \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

$$= \boxed{\{6, 8, 10, 12\}}$$

[3]

(c) $(N \cup R) \cap M = (\{1, 3, 5, 7, 9, 11, 13\} \cup \{0, 1, 2, 3, 4\}) \cap \{0, 2, 4, 6, 8\}$

$$= \{0, 1, 2, 3, 4, 5, 7, 9, 11, 13\} \cap \{0, 2, 4, 6, 8\}$$

$$= \boxed{\{0, 2, 4\}}$$

[5]

Question 2:

(a) Evaluate $\frac{15 \div 5 \cdot 4 \div 6 - 8}{-6 - (-5) - 8 \div 2} = \frac{2 - 8}{-6 + 5 - 4} = \frac{-6}{-5} = \boxed{\frac{6}{5}}$

[3]

(b) Evaluate the following expression if $p = -4$, $q = 8$ and $r = -10$:

$$\frac{\left(\frac{q}{4} - \frac{r}{5}\right)}{\left(\frac{p}{2} + \frac{q}{2}\right)} = \frac{\left(\frac{8}{4} - \frac{-10}{5}\right)}{\left(\frac{-4}{2} + \frac{8}{2}\right)}$$

$$= \frac{2 + 2}{-2 + 4}$$

$$= \frac{4}{2} = \boxed{2}$$

[3]

(c) Evaluate the following expression if $x = -4$ and $y = 2$:

$$\frac{|-8y + x|}{-|x|}$$

$$= \frac{|-8(2) + (-4)|}{-|-4|}$$

$$= \frac{|-16 - 4|}{-(4)}$$

$$= \frac{|-20|}{-4}$$

$$= \frac{20}{-4} = \boxed{-5}$$

[4]

Question 3:

(a) Simplify (assume the variables represent nonzero real numbers):

$$\begin{aligned} & \left(\frac{-5n^4}{r^2}\right)^3 \\ &= \frac{(-1)^3 5^3 (n^4)^3}{(r^2)^3} \\ &= \boxed{\frac{-125n^{12}}{r^6}} \end{aligned}$$

[2]

(b) Find the product:

$$\begin{aligned} & (r-3s+t)(2r-s+t) \\ &= 2r^2 - r\cancel{s} + r\check{t} - 6r\check{s} + 3\check{s}^2 - 3\check{s}t \\ & \quad + 2r\check{t} - \check{s}t + t^2 \\ &= \boxed{2r^2 + 3s^2 + t^2 - 7rs + 3rt - 4st} \end{aligned}$$

[3]

(c) Perform the division

$$\begin{array}{r} x^2 - 5 \\ x^2 + 0x + 1 \overline{) x^4 + 0x^3 - 4x^2 + 2x + 5} \\ \underline{-(x^4 + 0x^3 + x^2)} \\ -5x^2 + 2x + 5 \\ \underline{-(-5x^2 + 0x - 5)} \\ 2x + 10 \end{array}$$

$$= \boxed{x^2 - 5 + \frac{2(x+5)}{x^2 + 1}}$$

[5]

Question 4: Factor completely:

$$\begin{aligned}
 \text{(a)} \quad 8x^2 - 2x - 21 &= (8x^2 - 14x) + (12x - 21) \\
 &= 2x(4x - 7) + 3(4x - 7) \\
 &= \boxed{(2x + 3)(4x - 7)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 125x^3 - 27 &= (5x)^3 - 3^3 \\
 &= \boxed{(5x - 3)(25x^2 + 15x + 9)}
 \end{aligned}$$

[3]

$$\begin{aligned}
 \text{(c)} \quad 6p^4 + 7p^2 - 3 &= 6(p^2)^2 + 7(p^2) - 3 \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{let } w = p^2 \\
 &= 6w^2 + 7w - 3 \\
 &= 6w^2 + 9w - 2w - 3 \\
 &= 3w(2w + 3) - (2w + 3) \\
 &= (3w - 1)(2w + 3) \\
 &= \boxed{(3p^2 - 1)(2p^2 + 3)}
 \end{aligned}$$

[3]

[4]

Question 5:

(a) Write in lowest terms: $\frac{r^2 - r - 6}{r^2 + r - 12} = \frac{\cancel{(r-3)}(r+2)}{\cancel{(r-3)}(r+4)}$

$$= \boxed{\frac{r+2}{r+4}}$$

[4]

(b) Find the following product and write your answer in lowest terms:

$$\frac{x^2 + 2x - 15}{x^2 + 11x + 30} \cdot \frac{x^2 + 2x - 24}{x^2 - 8x + 15}$$

$$= \frac{(x-3)(x+5)}{(x+6)(x+5)} \cdot \frac{(x-4)(x+6)}{(x-3)(x-5)}$$

$$= \frac{\cancel{(x-3)}\cancel{(x+5)}(x-4)\cancel{(x+6)}}{\cancel{(x+6)}\cancel{(x+5)}\cancel{(x-3)}(x-5)}$$

$$= \boxed{\frac{(x-4)}{(x-5)}}$$

[6]