Section 4.1 Extra Exercises

Q1. Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is ***p dollars***, the revenue **R** (in dollars) is $R(p) = -4p^2 + 4000p$. What unit price should be established for the dryer to maximize revenue? What is the maximum revenue?

Ans: **p = \$500, Maximum revenue is \$1,000,000**

Q2. The John Deere Company has found that the revenue, in dollars, from sales of riding mowers is a function of the unit price ***p***, in dollars, that it charges. If the revenue ***R*** is

$$R(p) = -\frac{1}{2}p^2 + 1900p.$$

What unit price p should be charged to maximize revenue? What is the maximum revenue? **Ans: p = \$1,900** **Maximum revenue = \$1,805,000**

Q3. The marginal cost of a product can be thought of as the cost of producing one additional unit of output. For example, if the marginal cost of producing the 50th product is \$6.20, it cost \$6.20 to increase production from 49 to 50 units of output. Suppose the marginal cost ***C*** (in dollars) to produce x thousand MP3 players is given by the function $C(x) = x^2 - 140x + 7400$.

a) How many players should be produced to minimize the marginal cost?
b) What is the minimum marginal cost?

Ans: a) 70 wristwaches **b) \$2,500**

Q4. The marginal cost C (in dollars) of manufacturing ***x*** cell phones (in thousands) is given by $C(x) = 5x^2 - 200x + 4000$.

a) How many cell phones should be manufactured to minimize the marginal cost?
b) What is the minimum marginal cost?

Ans: a) 20 thousand cell phones manufactured b) \$2,000

Q5. The monthly revenue ***R*** achieved by selling ***x*** wristwaches is figured to be

$R(x) = 75x - 0.2x^2$. The monthly cost ***C*** of selling ***x*** wristwatches is $C(x) = 32x + 1750$.

a) How many wristwatches must the firm sell to maximize revenue? What is the maximum revenue?
b) Profit is given as $P(x) = R(x) - C(x)$. What is the profit function?
c) How many wristwatches must the firm sell to maximize profit? What is the maximum profit?

Ans: **a) Either 187 or 188 wristwatches. Maximum revenue is \$7,031.20**

b) $\mathbf{P(x) = -0.2x^2 + 43x - 1750}$

c) Either 107 or 108 wristwatches. Maximum profit is \$561.20.

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Chapter 4.1

Quadratic Function: $a(x-h)^2+k$ where vertex $=(h,k)$

1. In general the cost of construction per unit for an apartment complex will decrease as the number of units increase due to factors such as fixed survey costs and economies of scale on use of machinery and purchase of materials etc. However, after a certain number of units, the cost per unit increases due to less optimal designs to accommodate the extra units. In this way, the cost per unit is a function of x, the number of units being built and can be expressed as $c(x) = 2x^2 - 80x + 810$. Find the optimal number of units per complex for minimal cost and give the cost of construction for each unit.

Ans. $c(x)$ concave up with vertex (20,10). Therefore the answer is 20 units with a cost of 10 per unit.

2. A tour company has fixed assets (buses, fax machines, etc.) and a fixed number of employees. The cost of running a tour decreases as the number of tours increase due to better use of assets. However, after a certain number, the cost of running a tour increases due to factors such as less than optimal use of assets, overtime, etc. The cost of a tour is thus a function of the number of tours run, x, and is given by $c(x) = .04x^2 - .8x + 6$. Find the number of tours that result in the cheapest cost per tour.

Ans. $c(x)$ concave up with vertex (10,2). Therefore the answer is 10 tours.

3. $Profit = Revenue - Cost$ is, in many cases, a quadratic function with graph (a parabola) concave down; meaning that as x increases so does profit up to some value of x, after which the profit decreases with increased x. If profit is given by $p(x) = -2x^2 + 16x - 26$, find maximum profit and the value of x at which it occurs.

$P(x) = -2x^2 + 16x - 26$

Ans. $p(x)$ concave down with vertex (4,6). Therefore the answer is profit of 6 at $x = 4$.

4. A company does full restoration of automobiles of a particular make and year over a fixed time period. Let x be the amount of time that is spent on the painting. The value of the restored automobile is thus a function of x which can be expressed as $v(x) = -10x^2 + 24x + 10.6$. Find the amount of time that should be spent on the painting to maximize value and what the maximum value is.

Ans. $v(x)$ concave down with vertex (1.2,25). Therefore amount of time that should be spent on the painting would be 1.2 to give a maximum value of 25.

5. A software company is the sole producer of a certain game. As more games are produced revenues increase, as do costs but less so due to the benefits of economies of scale resulting in increased profits. However, after a certain number of games have been sold, the demand for the game begins to tail off and prices must be dropped, decreasing revenues and hence profits. If profit as function of x, the number of games produced, is given by $p(x) = -.000005x^2 + .02x - 15$, find the maximum profit that can be made and the number of games that should be produced to achieve it.

Ans. $p(x)$ concave down with vertex (2000,5). Therefore maximum profit is 5 when producing 2000 games.