Question 1:

(a) Find (i) the radius of convergence and (ii) the open interval of convergence for the power series

$$\sum_{k=1}^{\infty} \frac{(-1)^k k(x-1)^k}{5^k}$$

[5]

(b) Find (i) the radius of convergence and (ii) the open interval of convergence for the power series

$$\sum_{k=0}^{\infty} \frac{3^k x^k}{k!}$$

[5]

Question 2: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

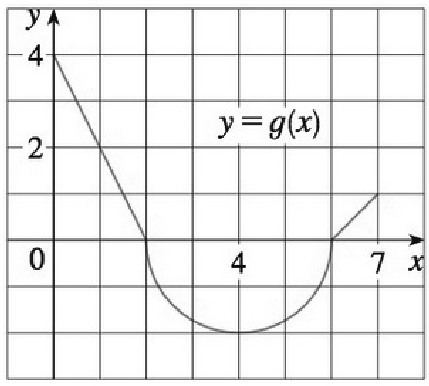
to evaluate

$$\int_0^2 (2 - x + 3x^2)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

[10]

Question 3: The graph of $y = g(x)$ over the interval $[0, 7]$ consists of two straight lines and a semicircle (half of a circle) as shown below. Determine $\int_0^7 g(x)\,dx$.



[5]

Question 4: The trunk of a growing tree has a circular cross-section with a radius that increases at a rate of $\frac{(5+t)}{1000}$ metres per year. If the trunk radius is currently $1/4$ m, what will be the radius in 20 years?

[5]

Question 5:

(a) Find $\int (\sin(x) - 2e^x + \pi)\ dx$

[2]

(b) Find $\int_1^4 \frac{1+\sqrt{x}}{x}\, dx$

[2]

(c) Find $\int_{-1}^1 x(1-x)^2\, dx$

[2]

Question 6: Find the value of c so that the average value of $f(x) = \sqrt{x}$ over $[0, 4]$ is equal to $f(c)$.

[4]

Question 7: Substitution Method:

(a) Find $\int x^2 e^{(x^3)}\, dx$

[3]

(b) Find $\int \frac{1}{x \ln(x)}\, dx$

[3]

(c) Find $\int_1^e \frac{\cos(\pi \ln(x))}{x}\, dx$

[4]