Question 1:

(a) Use a linear approximation $T_1(x)$ for $f(x) = \dfrac{1}{\sqrt{1+x}}$ to approximate $f(1/10)$. Express your answer as a single simplified fraction.

[5]

(b) Give an error bound for your approximation in part (a). Again, express your answer as a single simplified fraction.

[5]

Question 2:

(a) Find the Taylor polynomial of degree 2 for $f(x) = x^2 \ln(x)$ at $a = 1$.

[5]

(b) Suppose $T_2(x)$ in part (a) is used to approximate $f(4/5)$. Give an error bound on the approximation. Express your answer as a single simplified fraction. (Note: you are not being asked to find the approximation to $f(4/5)$ here, but only the error bound associated with the approximation.)

[5]

Question 3:

(a) Find the first four nonzero terms of the Maclaurin series for $f(x) = x^2 \cos(2x)$. You may leave the terms of your answer in factored form.

[5]

(b) Again for $f(x) = x^2 \cos(2x)$, determine $f^{(12)}(0)$. You may leave your answer as a fraction in factored form.

[5]

Question 4:

(a) Find the first four nonzero terms of the Maclaurin series for $f(x) = \dfrac{x^3}{1+x}$ and state the open interval of convergence.

[3]

(b) Find the Maclaurin polynomial of degree 17 for $g(x) = x^2 e^{-x^5}$

[3]

(c) Find the the first four nonzero terms of the Taylor series for $h(x) = \dfrac{1}{3-2x}$ about $a = 1$ and state the open interval of convergence of the series. (Hint: one way is to use the series for $\dfrac{1}{1-x}$.)

[4]

Question 5: Find the first three nonzero terms of the Maclaurin series for $f(x) = \arctan(x) \cdot e^x$

[5]

Question 6: Evaluate the following limit and state your answer as a single simplified fraction:

$$\lim_{x \to 0} \frac{\sin(x^2) - x^2 \cos(x^2)}{e^{(x^6)} - 1}$$

[5]