Question 1:

(a) Use the trapezoind rule on 3 subintervals to approximate $\int_0^3 e^{x(1-x)(2-x)/6}\,dx$. (Simplify your final answer.)

[5]

(b) The second derivative of $f(x) = e^{x(1-x)(2-x)/6}$ can be shown to be between -1 and 15 on the interval $[0, 3]$. Use this information to determine an error bound $|E_{T_3}|$ on your approximation in part (a). Simplify your final answer.

(Recall: the error in using the trapezoid rule to approximate $\int_a^b f(x)\,dx$ using n subintervals is at most $\frac{K(b-a)^3}{12n^2}$ where $|f''(x)| \leq K$ on $[a, b]$.)

[5]

Question 2:

(a) Determine whether the following improper integral is convergent or divergent. If convergent, evaluate the integral. Make proper use of any required limits:

$$\int_{-1}^{1} \frac{e^x}{e^x - 1}\, dx$$

[5]

(b) Use the comparison theorem to determine whether the following integral is convergent or divergent:

$$\int_{1}^{\infty} \frac{1}{\sqrt{x}(1+x)}\, dx$$

[5]

Question 3: The region bounded by the curves $y = x^2$ and $y = 1$ is divided into two regions by the line $y = b$. Find b if the two regions have equal areas.

[5]

Question 4: The base of the solid S is the triangular region in the xy-plane with vertices at $(0, 0)$, $(1, 0)$ and $(0, 1)$. Cross-sections perpendicular to the y-axis are squares. Determine the volume of S.

[5]

Question 5: The region in the first quadrant bounded between $y = 1 - x^2$ and $y = 1 - x$ is rotated about the horizontal line $y = 1$. Determine the volume of the resulting solid (both the washer and cylindrical shells methods can be used here.)

[10]

Question 6: The region bounded by the curves $y = e^x$, $y = x$, $x = 0$ and $x = 1$ is rotated about the vertical line $x = -2$. Determine the volume of the resulting solid.

[10]