

# Method of Undetermined Coefficients

Use to solve

$$L(y) = g(x)$$

where

$$L = a_n D^n + a_{n-1} D^{n-1} + \cdots + a_1 D + a_0 .$$

First, determine the complementary function  $y_c$  which is the general solution to

$$L(y) = 0 .$$

Next, *guess* a particular solution  $y_p$  based on the form of  $g(x)$ . Consider  $y_p$  as a sum of trial functions determined as follows:

If $g(x)$ contains a term which is a constant multiple of ...	trial function to appear as a term of $y_p^*$
$k$ (a constant)	$A$
$x^m$	$c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0$
$e^{cx}$	$Ae^{cx}$
$\sin(px + q)$	$A \sin(px + q) + B \cos(px + q)$
$\cos(px + q)$	$A \sin(px + q) + B \cos(px + q)$
$x^m e^{cx}$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0) e^{cx}$
$x^m \sin(px + q)$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0) \sin(px + q) + (d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0) \cos(px + q)$
$x^m \cos(px + q)$	$(c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0) \sin(px + q) + (d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0) \cos(px + q)$
$e^{cx} x^m \sin(px + q)$	$e^{cx} (c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0) \sin(px + q) + e^{cx} (d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0) \cos(px + q)$
$e^{cx} x^m \cos(px + q)$	$e^{cx} (c_m x^m + c_{m-1} x^{m-1} + \cdots + c_1 x + c_0) \sin(px + q) + e^{cx} (d_m x^m + d_{m-1} x^{m-1} + \cdots + d_1 x + d_0) \cos(px + q)$

\* If a constant multiple of a term of the trial function is already a term of the complementary solution  $y_c$ , multiply that trial function by  $x^j$  where  $j$  is the smallest natural number such that the new trial function no longer duplicates a term of  $y_c$ .