

Question 1:

(a) Given that $y_1(x) = e^x$ is one solution of

$$xy'' - (x + 1)y' + y = 0, \quad x > 0,$$

use reduction of order to find a second linearly independent solution $y_2(x)$. (Do not use "the formula", but rather, find $y_2(x)$ from first principles. You may check your result using the formula however.)

[7]

(b) Show that $y_1(x)$ and $y_2(x)$ from part (a) are linearly independent.

[3]

Question 2:

(a) Find the general solution of $y'' - 2y' + y = 0$

[3]

(b) Suppose you are finding the general solution of $y'' - 2y' + y = x^2 e^x$ using the method of undetermined coefficients. Making use of your result in part (a), state the form of the trial solution y_p . (Note: do not determine the constants in y_p , simply state the form you would use.)

[3]

Question 3: Calculate $\mathcal{L}\{f(t)\}$ where $f(t) = te^{2t}$ (do not use tables).

[4]

Question 4: Find the general solution of

$$x^2 y'' + xy' - y = \ln(x)$$

Question 5: A mass of 1 slug, when attached to a spring, stretches it 2 feet then comes to rest at equilibrium. Starting at $t = 0$ and external force of $f(t) = 8 \sin(4t)$ is applied to the system. The surrounding medium imparts a damping force equal to 8 times the instantaneous velocity. Find the equation of motion. (Use any method you like to solve the resulting differential equation.)

Question 6: Solve using the Laplace transform:

$$y'' + 4y = 2e^{-t}, \quad y(0) = 1, \quad y'(0) = 0$$