Question 1 [10]: For this question use the differential equation $\dfrac{dv}{dt} = 3 - 2v^2$

(a) Determine (i) the equilibrium solutions, (ii) sketch the one dimensional phase portrait, and (iii) classify each equilibrium solution as asymptotically stable, unstable or semi-stable.

[4]

(b) If $v(0) = -1$, what is $\lim_{t\to\infty} v(t)$?

[2]

(c) If $v(0) = 2$, use Euler's method with $h = 0.05$ to approximate $v(0.15)$ to three decimals.

[4]

Question 2: Solve the IVP $\dfrac{dy}{dx} = 8x^3e^{-2y}, \quad y(1) = 0$. State the solution in explicit form and give the largest interval on which the solution is valid.

[5]

Question 3: Solve the IVP $t^2\dfrac{dx}{dt} + 3tx = \dfrac{\ln(t)}{t^2} + 1, \quad x(1) = 0$. State the solution in explicit form and give the largest interval on which the solution is valid.

[5]

Question 4: Solve: $(ye^{xy} - \frac{1}{y})\,dx + (xe^{xy} + \frac{x}{y^2})\,dy = 0$. You may leave your solution in implicit form.

[5]

Question 5: Solve: $\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$, and also state constant solutions, if any. You may leave your solution in implicit form.

[5]

Question 6: Solve the IVP $\dfrac{dy}{dx} = 1 + e^{y-x+5}, \quad y(5) = 0$. State your solution in explicit form.

[5]

Question 7: Determine the most general form of the function $M(x, y)$ if the following equation is exact:

$$M(x, y)\, dx + (\sin(x)\cos(y) - xy - e^{-y})\, dy = 0$$

[5]

Question 8 [10]: The Gompertz differential equation

$$\frac{dP}{dt} = P(a - b \ln P)$$

(with a and b constant) is a modification of the logistic differential equation used for population growth.

(a) Find an explicit solution to this differential equation if a and b are both positive and $P(0) = 1$.

[6]

(b) Will the population ever exceed its initial value of $P(0) = 1$? Explain.

[2]

(c) What is $\lim_{t \to \infty} P(t)$?

[2]