Question 1:

(a) Given that $y_1(x) = 1/x$ is one solution of

$$2x^2y'' + 3xy' - y = 0, \qquad x > 0,$$

use reduction of order to find a second linearly independent solution $y_2(x)$. (Do not use "the formula", but rather, find $y_2(x)$ from first principles. You may check your result using the formula however.)

[6]

(b) Show that $y_1(x)$ and $y_2(x)$ from part (a) are linearly independent.

[4]

Question 2: Find the general solution for each of the following:

(a) $y'' + y' + y = 0$

[3]

(b) $y'' + 2y' + y = 0$

[3]

(c) $2y'' + \frac{5}{x}y' + \frac{y}{x^2} = 0$

[4]

Question 3:

(a) Solve the following initial value problem:

$$y'' - 5y' + 4y = 0, \qquad y(0) = 12, y'(0) = -3$$

[5]

(b) Use part 2(b) to find the general solution of

$$y'' + 2y' + y = e^{-x} \ln(x)$$

[5]

Question 4: A 1 kg mass is attached to a spring whose constant is $k = 16$ N/m. The entire spring-mass system is then submerged in a liquid that imparts a damping force of $\beta = 10$ times the velocity of the mass. At time $t = 0$ the mass is released from a point 1 m below equilibrium with an initial velocity of 12 m/s upward. Set up and solve the differential equation describing the position $x(t)$ of the mass for $t \geq 0$.

[10]

Question 5: The temperature $u(r)$ between concentric spheres of radius $r = a$ and $r = b$, (where $a < b$) is determined by the boundary value problem

$$r\frac{d^2u}{dr^2} + 2\frac{du}{dr} = 0, \qquad u(a) = u_0, u(b) = u_1,$$

where u_0 and u_1 are constants. Solve for $u(r)$.

[10]

Question 6:

(a) Calculate $\mathcal{L}\{f(t)\}$ where $f(t) = te^{-2t-3}$ (do not use tables)

[3]

(b) Determine $\mathcal{L}^{-1}\{F(s)\}$ where $F(s) = (s^2 + s - 20)^{-1}$

[3]

(c) Solve using the Laplace transform:

$$y'' + 9y = e^t, \quad y(0) = 0, \quad y'(0) = 0$$

[4]