1. Find the Laplace transform of $f(t)$ where $f(t) = t$ for $0 \leq t < 2$ and $f(t+2) = f(t)$.

2. Solve using the Laplace transform:

$$y'' + 9y = 34e^{-5t} + 9\delta(t-5), \quad y(0) = 0, \quad y'(0) = -1$$

3. Solve using the Laplace transform:

$$y'' + 4y' + 5y = \delta(t-a), \quad y(0) = 0, \quad y'(0) = 0, \quad a > 0$$

4. Use the convolution product to find

$$\text{(a)} \;\; \mathcal{L}^{-1}\left\{\frac{1}{(s+3)(s^2+1)}\right\} \qquad \text{(b)} \;\; \mathcal{L}^{-1}\left\{\frac{3s^2}{(s^2+1)^2}\right\}$$

5. For $x > 0$ the gamma function $\Gamma(x)$ is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$$

   (a) Show that $\Gamma(1) = 1$ and that $\Gamma(x+1) = x\Gamma(x)$. (This shows that the gamma function generalizes the factorial function, with $\Gamma(n+1) = n!$ for integer $n \geq 0$)

   (b) Show that for $\alpha > -1$

$$\mathcal{L}\{t^\alpha\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$$

   (c) Using methods from multivariable calculus (specifically, double integrals in polar coordinates) it can be shown that $\Gamma(1/2) = \sqrt{\pi}$. Use this fact to determine

$$\mathcal{L}\{t^{-1/2} + 2t^{3/2} + 8t^{5/2}\}$$

   Your final answer should not contain gamma functions.