

1. Find the general solution to

$$y'' + 9y = \sec^2(3t)$$

2. Find the general solution to

$$y'' + 4y' + 4y = e^{-2t} \ln(9t)$$

3. Find the general solution on $(0, \infty)$ to

$$y'' + \left(\frac{6}{t}\right)y' + \left(\frac{4}{t^2}\right)y = 0$$

4. Solve the initial value problem:

$$t^2 y'' - t y' + 5y = 0, \quad y(1) = 2, \quad y'(1) = -2$$

5. A mass weighing 10 lbs stretches a spring $1/4$ of a foot. This mass is removed and replaced with a mass of 1.6 slugs. The new mass is released from a point $1/3$ of a foot above equilibrium with a downward initial velocity of $5/4$ ft/s.

(a) Set up and solve the differential equation for the equation of motion of the mass. Express your answer in the form $y = A \sin(\omega t + \phi)$.

(b) Determine the first time $t > 0$ at which the mass reaches a distance of $1/2$ of the amplitude below equilibrium.

6. A 4 foot spring stretches an additional 4 feet after a mass weighing 8 lbs is attached to it. The mass moves through a medium which imparts a damping force equal to $\sqrt{2}$ times the instantaneous velocity of the mass. The mass is initially released from equilibrium with an initial velocity of 5 ft/s downward.

(a) Set up and solve the differential equation for the equation of motion of the mass.

(b) Determine the time $t > 0$ at which the mass reaches its extreme displacement.

(c) What is the displacement at the extreme?