

1. For each of the following differential equations, state (i) the order and (ii) whether the equation is linear or non linear:

(a) $5\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 9x = 2\cos(3t)$

(b) $\frac{dy}{dx} = \frac{y(2-3x)}{x(1-3y)}$

(c) $\frac{dx}{dt} = k(4-x)(1-x)$ where k is a constant.

(d) $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

2. Determine whether the given function is a solution to the given differential equation:

(a) $y = \sin(x) + x^2$, $\frac{d^2y}{dx^2} + y = x^2 + 2$

(b) $\theta = 2e^{3t} - e^{2t}$, $\frac{d^2\theta}{dt^2} - \theta\frac{d\theta}{dt} + 3\theta = -2e^{2t}$

3. Determine all values of m for which $\phi(x) = e^{mx}$ is a solution to $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$.

4. Determine all values of m for which $\phi(x) = x^m$ is a solution to $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 5y = 0$.

5. Show that $\phi(x) = c_1e^x + c_2e^{-2x}$ is a solution to

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Determine c_1 and c_2 if

(a) $y(0) = 2$ and $y'(0) = 1$

(b) $y(1) = 1$ and $y'(1) = 0$

6. Textbook exercise 1.3.10

7. For the autonomous differential equation

$$\frac{dy}{dx} = \frac{ye^y - 9y}{e^y}$$

- (a) Find the critical points.
 (b) Sketch the one-dimensional phase portrait.
 (c) Classify the critical points as asymptotically stable, unstable, or semi-stable.