Question 1: Use the Wronskian to determine if $S = \left\{x, e^{-x}, xe^{x}\right\}$ is a linearly independent set on $(-\infty, \infty)$.

[5]

Question 2: Determine whether $S = \{(1,0,0,0),(1,1,1,1),(1,0,1,0),(0,1,0,1)\}$ is a basis for $\mathbb{R}^4$.

[5]

Question 3: Let $\mathbf{p}_1 = 1 + x$, $\mathbf{p}_2 = 1 + x^2$ and $\mathbf{p}_3 = x + x^2$. If $\mathbf{p} = 2 - x + x^2$ and $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is a basis, find $(\mathbf{p})_S$. (Recall, $(\mathbf{p})_S$ is the coordinate vector for $\mathbf{p}$ relative to the basis S.)

[5]

Question 4: Find a basis and state the dimension of the row space of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -2 \\ 4 & 3 & -1 \\ 6 & 5 & 1 \end{bmatrix}$$

[5]

Question 5: Find a basis and state the dimension for the subspace of $\mathbb{R}^4$ consisting of all vectors (a, b, c, d) where $b = a + c$ and $d = -2a + 3c$.

[5]

Question 6: Find a basis for the subspace of R^3 spanned by $\mathbf{v}_1 = (1, -1, 3)$, $\mathbf{v}_2 = (5, -4, -4)$ and $\mathbf{v}_3 = (7, -6, 2)$.

[5]

Question 7: The matrix $\mathbf{A} = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & -2 \end{bmatrix}$ has REF $\mathbf{R} = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Give a basis and state the dimension of the row space of $\mathbf{A}$.

[2]

(b) Give a basis and state the dimension of the column space of $\mathbf{A}$.

[2]

(c) Give a basis and state the dimension of the null space of $\mathbf{A}$. (Equivalently, give a basis and state the dimension of the solution space of $\mathbf{Ax} = \mathbf{0}$.)

[3]

(d) Determine rank($\mathbf{A}$) and nullity($\mathbf{A}$) .

[1]

(e) Determine rank($\mathbf{A}^T$) and nullity($\mathbf{A}^T$) .

[1]

Question 8: Let the transformation T_1 represent projection onto the xy-plane and T_2 be the transformation which rotates a vector in R^3 counter-clockwise about the y-axis by an angle θ.

(a) Determine the standard matrix **A** for the transformation T_1 .

[2]

(b) Determine the standard matrix **B** for the transformation T_2 .

[3]

(c) Determine the image of the line segment joining $(2, 1, 1)$ and $(1, 3, -1)$ if it is first rotated counter-clockwise about the y-axis by $\pi/3$ (or $60°$) and then projected onto the xy-plane.

[3]

(d) Is applying the projection followed by the rotation the same as applying the rotation followed by the projection? Explain using matrices.

[2]